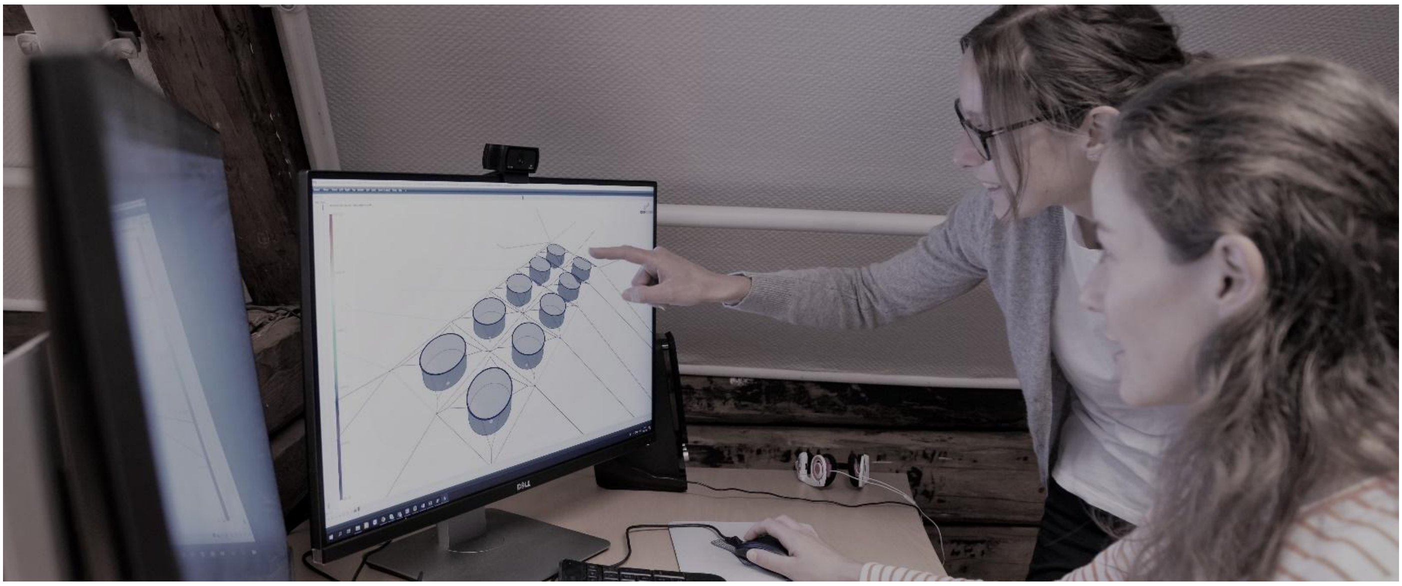
AquaSim training courses

- **Hydrodynamic analysis in AquaSim using "MacCamy-Fuchs"**



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Content

1 Prerequisites

The tutorial presents a simple case study with the purpose of demonstrating functionality in AquaSim.

It is assumed that the user is familiar with the basic principles of modelling and specifying material parameters in AquaEdit, as well as conducting analyses. If you are looking for an introduction to AquaSim we advise you to start with the Basic program tutorials. Furthermore, it is assumed that the user has some experience with linear potential flow theory, and analytical solutions to the “diffraction problem”.

2 Introduction

This tutorial introduces how wave excitation forces are calculated analytically in AquaSim based on the theory first described in (Maccamy & Fuchs, 1954). Furthermore, you will be introduced to how radiation forces (hydrodynamic damping and added mass) are calculated, when using the “MacCamy-Fuchs” load formulation.

The purpose is to introduce the “MacCamy-Fuchs” load formulation, the theoretical background, the validity of this load formulation, how to initiate this load formulation in AquaSim, as well as comparing the results from the “MacCamy-Fuchs” load formulation with numerical results from the NEMOH solver using the load formulation “Numerical diffraction” and the analytical long wave approximation (Morison).

3 Theoretical background MacCamy-Fuchs

3.1 Validity

The analytical solution for calculating the wave excitation pressure and forces, described in (Maccamy & Fuchs, 1954), is based on linear potential flow theory and assumes an infinitely long vertically oriented circular cylinder with a fixed radius. Meaning that infinite water depth is assumed (deep-water waves). Therefore, care should be taken, when using this load formulation for analysing anything other than what is assumed for this load formulation.

Furthermore, if you are using the “MacCamy-Fuchs” load formulation to analyse a vertical cylinder with fixed radius and finite length, one should be careful with respect to evaluating which wave periods that one might consider valid for the structure under consideration.

A rule of thumb will be to assume that the load formulation will provide sufficiently accurate results for a wave period (T) corresponding to a wavelength (λ) of twice the length of the cylinder (L) or less, i.e. $\lambda \leq 2L$, where $\lambda = \frac{g}{2\pi}T^2$ for deep-water waves. For example, given a cylinder with fixed radius and a length $L = 110m$, this load formulation should at least be valid for wave lengths $\lambda \leq 220m$, corresponding to wave periods of $T \leq 11.9s$ assuming deep-water waves.

In general care should be taken when analyzing structures with the “MacCamy-Fuchs” load formulation where diffraction is of importance, meaning for wavelengths shorter than 5 times the diameter of the cylinder (D), i.e. $\lambda < 5 * D$, as well as when the wavelength is greater than twice the length of the cylinder, i.e. $\lambda \geq 2L$. Typically, this corresponds to short cylinders with large diameters.

3.2 Total wave excitation pressure

The total wave excitation pressure using the “MacCamy-Fuchs” load formulation in AquaSim is calculated as described by Equation 1 - Equation 6. Abbreviations are given in Table 1. For more details, see (Maccamy & Fuchs, 1954) and (Aquastructures AS, 2024a).

Table 1 Abbreviations for parameters in the “MacCamy-Fuchs” load formulation.

Abbreviation	Description	Unit
ρ	Density of water, 1025	kg/m^3
g	Gravitational acceleration, 9.81	m/s^2
ζ	Wave amplitude	m
k	Wave number	1/m
z	Vertical position, 0 means still water level. Positive upwards.	m
h	Depth of sea bottom	m
i	Complex unit (0,1)	-
B_n	Coefficient, see Equation 2	-
H_n	Hankel function, first kind	-
J'_n	Bessel function, derivative	-
ε_n	ε_0= 1, else 2	-
ω	Wave frequency	rad/s
t	Time	s

The diffraction pressure from “MacCamy-Fuchs” is calculated as:

$$p_{MF} = \rho g \zeta \frac{\cosh k(z+h)}{\cosh kh} \sum_{n=0}^{\infty} i\left[B_n {H_n}^1(kr)\right] \cos n\theta e^{-i\omega t}$$

Equation 1

Where:

$$B_n = -\varepsilon_n i^n \frac{J_n^{\cdot}(kr)}{H_n^{(1)\cdot}(kr)}$$

Equation 2

The Frode-Kriloff pressure in a regular sea with airy waves, from the incident wave is found as:

$$p_{FC} = \rho g \zeta \frac{\cosh k(z+h)}{\cosh kh} \sin(\omega t - kx)$$

Equation 3

If irregular waves are considered, the pressure from the diffracted wave field on the surface of the structure is found as:

$$p_{MF} = \sum_{m=1}^{N} \rho g \zeta_m \frac{\cosh k_m(z+h)}{\cosh k_m h} \sum_{n=0}^{\infty} i\left[B_n {H_n}^1(kr)\right] \cos n\theta e^{-i\omega t + \varepsilon_n}$$

Equation 4

While the Frode-Kriloff pressure for an irregular sea is found as:

$$p_{FC} = \sum_{n=1}^{N} \rho g \zeta_n \frac{\cosh k_n(z+h)}{\cosh k_n h} \sin(\omega_n t - k_n x + \varepsilon_n)$$

Equation 5

The total pressure at a given point is then found as:

$$p = p_{FC} + p_{MF}$$

Equation 6

Since the "MacCamy-Fuchs" theory for diffracted waves is only valid for vertical cylinders, the pressure from the diffracted wave field p_{MF} is multiplied with the vertical projection of the area.

3.3 Added mass and hydrodynamic damping

Both the added mass and hydrodynamic damping are calculated in a simplified manner when using the "MacCamy-Fuchs" load formulation and relates these parameters to the geometry and the volume of the structure and are therefore also frequency independent, using this load formulation. The reason being that the "MacCamy-Fuchs" load formulation only analytically solves the "diffraction problem" and not the "radiation problem". This methodology is identical to the methodology for calculating the dynamic inner fluid mass, and detailed descriptions and figures can be found in (Aquastructures AS, 2025c).

3.3.1 Vertically oriented panels

For vertically oriented panels with otherwise arbitrary orientation, the horizontal (normal) added mass and hydrodynamic damping per m^2 are calculated as follows:

$$Added\ mass\ per\ m^2, horizontal = R * C_{Amass_{hor}} * \rho * \hat{\boldsymbol{r}}_{\boldsymbol{hor}} \cdot \hat{\boldsymbol{n}}_{\boldsymbol{hor}}$$

Equation 7

$$Hydrodynamic\ damping\ per\ m^2, horizontal = R * C_{Hdamp_{hor}} * \rho * \hat{\boldsymbol{r}}_{\boldsymbol{hor}} \cdot \hat{\boldsymbol{n}}_{\boldsymbol{hor}}$$

Equation 8

Here R is the distance from the panel to the geometric centerline of the 2D volume of the structure at the vertical position of the panel as seen in Figure 1. $C_{Amass_{hor}}$ and $C_{Hdamp_{hor}}$ are the horizontal coefficients for added mass and hydrodynamic damping, as highlighted in yellow in Figure 2. The vectors $\hat{\boldsymbol{r}}_{\boldsymbol{hor}}$ and $\hat{\boldsymbol{n}}_{\boldsymbol{hor}}$ are the normalized horizontal position vector of the panel and the normalized horizontal normal vector of the panel respectively. Furthermore, ρ is the density of seawater (1025 kg/m^3).

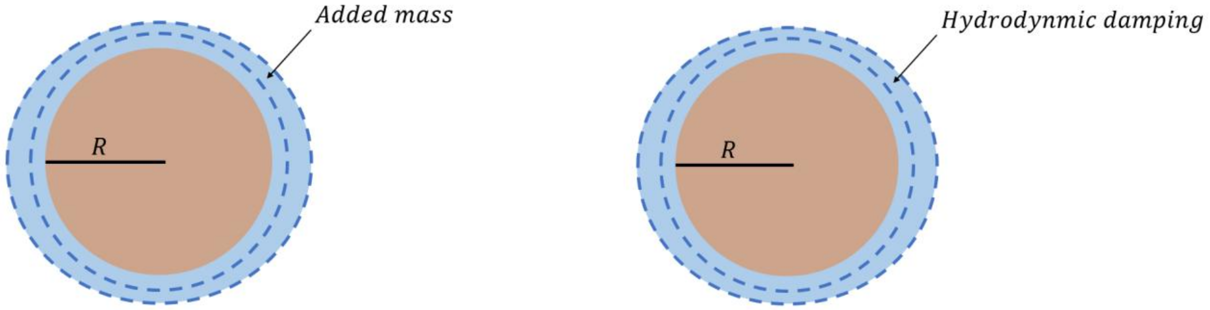

Figure 1 Illustration of the distance R.

3.3.2 Horizontally oriented panels

For horizontally oriented panels with otherwise arbitrary orientation, the vertical (normal) added mass and hydrodynamic damping per m^2 are calculated as follows:

$$Added\ mass\ per\ m^2, vertical = z_{pos} * C_{Amass_{vert}} * \rho$$

Equation 9

$$Hydrodynamic\ damping\ per\ m^2, vertical = z_{pos} * C_{Hdamp_{vert}} * \rho$$

Equation 10

$C_{Amass_{vert}}$ and $C_{Hdamp_{vert}}$ are the vertical coefficients for added mass and hydrodynamic damping, as highlighted in green in Figure 2.

3.3.3 Arbitrary oriented panels

For arbitrary oriented panels the added mass and hydrodynamic damping per m^2, acting normally on the panel, are calculated as follows:

$$Added\ mass\ per\ m^2 = \rho(z_{pos} * \sqrt{n_z^2} * C_{Amass_{vert}} + R * \hat{\boldsymbol{r}}_{\boldsymbol{hor}} \cdot \hat{\boldsymbol{n}}_{\boldsymbol{hor}} \sqrt{1 - n_z^2} * C_{Amass_{hor}})$$

Equation 11

$$Hydrodynamic\ damping\ per\ m^2 = \rho(z_{pos} * \sqrt{n_z^2} * C_{Hdamp_{vert}} + R * \hat{\boldsymbol{r}}_{\boldsymbol{hor}} \cdot \hat{\boldsymbol{n}}_{\boldsymbol{hor}} \sqrt{1 - n_z^2} * C_{Hdamp_{hor}})$$

Equation 12

Here $\sqrt{n_z^2}$ and $\sqrt{1 - n_z^2}$ are the absolute values of the vertical and horizontal component of the unit normal vector of the panel, respectively.

3.3.4 Other notes on added mass and hydrodynamic damping

Because of this formulation, the total vertical and total horizontal added mass and hydrodynamic damping are directly proportional to the total 3D volume of the structure, as long as the structure either consists strictly of horizontal and vertical panels (e.g. cylinders, box shapes etc.) or has radial symmetry (e.g. spheres, half spheres etc.).

Note 1: The exact same methodology is applied for calculating the added mass and hydrodynamic damping when using the load formulation "Flexible tarp".

Note 2: The added mass coefficients $C_{Amass_{hor}}$ and $C_{Amass_{vert}}$ are unitless. The hydrodynamic damping coefficients $C_{Hdamp_{hor}}$ and $C_{Hdamp_{vert}}$ have unit [1/s], when using the "Flexible tarp" and "MacCamy-Fuchs" load formulations, but are unitless when using the "Numerical diffraction" load formulation.

⊟ **Added mass and damping**	
Added mass coefficient horizontal	1.0
Added mass coefficient vertical	1.0
Added mass indicator	0: Mean free surface
Hydrodynamic damping coefficient horizontal	1.0
Hydrodynamic damping coefficient vertical	1.0
Damping coefficient (flexible tarp)	0.0
Damping coefficient (flexible tarp) tangential to panels	0.0

Figure 2 Added mass and hydrodynamic damping coefficients.

4 How to initiate the "MacCamy-Fuchs" load formulation in AquaSim

There are mainly two different ways to initiate the "MacCamy-Fuchs" load formulation in AquaSim, and both requires that the component group, representing the geometry and elements you want to calculate the hydrodynamic properties of, is either set to element type "Membrane" or "Membrane X".

The next step is then to go to the dropdown menu for "Load Formulation" and choose either "Lice skirt" or "Closed compartment" as shown in Figure 3. You then go to the dropdown menu for "Wave excitation load" and choose "MacCamy-Fuchs", as shown in Figure 4.

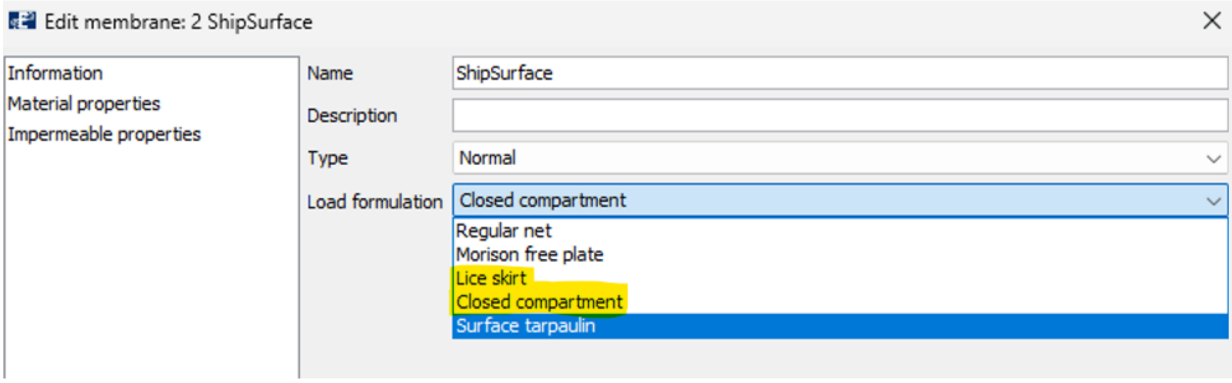

Figure 3 Load formulations that can be used to initiate the MacCamy-Fuchs in AquaSim, highlighted in yellow.

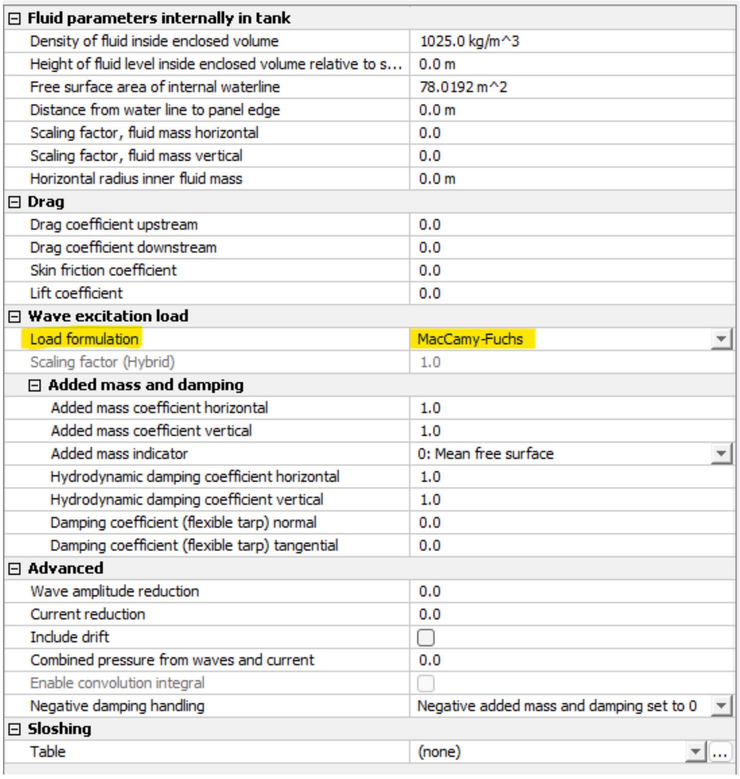

Figure 4 How to initiate MacCamy-Fuchs in AquaSim using "Lice skirt" or "Closed compartment".

5 Analysis model

There has been performed analysis of a vertical cylinder modelled with “Membrane” elements as shown in Figure 5, with normal vectors pointing inwards and with parameters as described in Table 2 and Figure 8, previously used in (Aquastructures AS, 2025b). The diagonal of the “Membrane” elements has a length of $1.34m$, meaning that one can expect sufficiently accurate results for wave periods $T \geq 2.3s$, when using “Numerical diffraction”, according to the methodology described in (Aquastructures AS, 2025a).

Furthermore, there are modelled a frame consisting of “Beam” elements along the vertices of the “Membrane” elements, a vertical centre beam and eight symmetrical horizontal radial beams at the top of the cylinder, as shown in Figure 6, with parameters as presented in Figure 7.

There is also modelled a “Truss” element at the top of the cylinder, as seen in Figure 5, with properties as shown in Figure 9.

In the analysis model, all nodes are restrained to only be able to move in the global x-direction (along red axis), except the node at the end of the horizontal truss element at the top of the cylinder, which is restrained in all degrees of freedom (DOF).

Table 2 Description of analysis model.

Description	Value	Unit
Length, ***L***	110	m
Diameter, ***D***	10	m
“Membrane” elements, circumferentially	32	#
“Membrane” elements, longitudinally	110	#
“Membrane” elements, total	4032	#
Wave amplitude, ***ζ***	1.0	m
Wave period, ***T***	See Figure 10	s
Wave direction	0 (along positive x-axis, i.e. red axis in Figure 5)	deg
Number of steps per wave period [-]	160	#

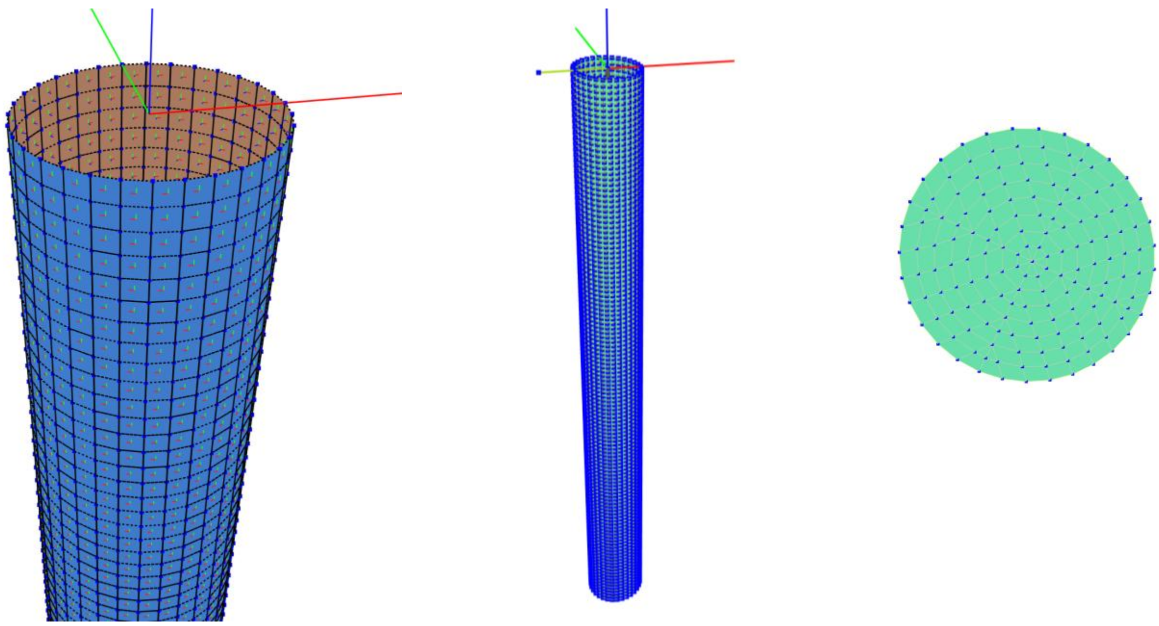

Figure 5 Analysis model of vertical cylinder with a length of 110m and diameter of 10m. Illustration of orientation of normal vectors to the left, full model in the middle and bottom of model to the right.

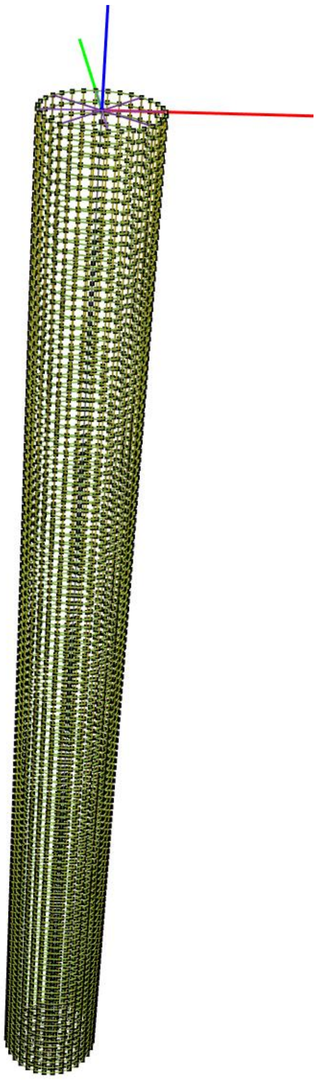

Figure 6 Frame consisting of "Beam" elements to ensure structural stiffness.

Material properties	
E-modulus	2.1E11 N/m^2
G-modulus	8.08E10 N/m^2
Cross sectional properties	
Area	6.4E-3 m^2
Iy	7.2533E-6 m^4
Iz	7.2533E-6 m^4
It	1.024E-5 m^4
Weight and volume per meter length	
Volume	0.0 m^3/m
Mass density	0.0 kg/m^3
Weight in air	0.0 kg/m
Weight in water	0.0 kg/m
Advanced	
Rayleigh damping (mass)	0.0
Rayleigh damping (stiffness)	0.0
Mass radius	0.0 m
Pretension	0.0

Drag load	
Drag coefficients	
Y	0.0
Z	0.0
Added mass coefficients	
Cay	0.0
Caz	0.0
Diameter for drag	
Y (depth)	0.0 m
Z (width)	0.0 m
Wave generated damping coefficient	
Horizontal motion	0.0
Vertical motion	0.0
Rotation	0.0
Advanced	
Slamming shape	Circle
Wave amplitude reduction	0.0
Current reduction	0.0
Longitudinal drag coefficient	0.0
Wind load	

Figure 7 Properties of "Beam" elements in frame.

Thickness (for impermeable)	
Thickness Y	1.2566E-7 m
Thickness Z	1.2566E-7 m
Thickness	2.5133E-7 m
Properties	
E-module	10.0 N/m^2
Thread diameter	2E-3 m
☐ Area	3.1416E-6 m^2
Mass density	0.0 kg/m^3
☑ Relative density in water	0.0 kg/m^3
No compression forces	☐
Solidity	
Pretension Y	0.0
Pretension Z	0.0
Growth coefficient	1.0
Maskwidth Y	25.0 m
Maskwidth Z	25.0 m
Solidity	0.016 %
Solidity incl growth	0.016 %
Advanced	
Rayleigh damping stiffness	0.0
Rayleigh damping mass	0.0
Non-linear data	(none)

Fluid parameters internally in tank	
Density of fluid inside enclosed volume	1025.0 kg/m^3
Height of fluid level inside enclosed volume relative to s…	0.0 m
Free surface area of internal waterline	78.0192 m^2
Distance from water line to panel edge	0.0 m
Scaling factor, fluid mass horizontal	0.0
Scaling factor, fluid mass vertical	0.0
Horizontal radius inner fluid mass	0.0 m
Drag	
Drag coefficient upstream	0.0
Drag coefficient downstream	0.0
Skin friction coefficient	0.0
Lift coefficient	0.0
Wave excitation load	
Load formulation	MacCamy-Fuchs
Scaling factor (Hybrid)	1.0
Added mass and damping	
Added mass coefficient horizontal	1.0
Added mass coefficient vertical	1.0
Added mass indicator	0: Mean free surface
Hydrodynamic damping coefficient horizontal	1.0
Hydrodynamic damping coefficient vertical	1.0
Damping coefficient (flexible tarp) normal	0.0
Damping coefficient (flexible tarp) tangential	0.0
Advanced	
Wave amplitude reduction	0.0
Current reduction	0.0
Include drift	☐
Combined pressure from waves and current	0.0
Enable convolution integral	☐
Negative damping handling	Negative added mass and damping set to 0
Sloshing	
Table	(none)

Figure 8 Material properties of "Membrane" elements corresponding to the structure surface.

Information	
Name	Kraftmåler øvre
Description	
Properties	
E-modulus	1E11 N/m^2
Area	1E-3 m^2
Diameter	0.035682 m
☐ Volume	1E-3 m^3/m
Mass density	0.0 kg/m^3
Weight in air	0.0 kg/m
☑ Weight in water	0.0 kg/m
Drag loads	
Diameter Y	0.0 m
Diameter Z	0.0 m
Drag coefficient Y	0.0
Drag coefficient Z	0.0
Longitudinal	
Drag coefficient X	0.0
Added mass	
Added mass coefficient Y	0.0
Added mass coefficient Z	0.0
Longitudinal	
Added mass coefficient X	0.0
Advanced	
No compression forces	☐
Pretension	0.0
Breaking load	0.0 N
Material coefficient	0.0
Rayleigh dampening (mass)	0.0
Rayleigh dampening (stiffness)	0.0

Figure 9 Properties of "Truss" in analysis model.

6 Load cases, analysis results and verification

6.1 Load cases

There are performed analyses with waves with amplitude of 1.0m and periods ranging from 3.0s to 100.0s respectively, as shown in Figure 10. The waves move in the direction of the positive x-axis, i.e. red axis in Figure 5.

☑	Nr	SysH[deg]	Hs[m]	T[s]	WaveH[d...	Vc[m/s]	CurrDir[d...	U10[m/s]	Comment	Group
☑	1	0.0	1.053	3	180.0	0	0	0		01
☑	2	0.0	1.053	4	180.0	0	0	0		02
☑	3	0.0	1.053	5	180.0	0	0	0		03
☑	4	0.0	1.053	6	180.0	0	0	0		04
☑	5	0.0	1.053	7	180.0	0	0	0		05
☑	6	0.0	1.053	8	180.0	0	0	0		06
☑	7	0.0	1.053	9	180.0	0	0	0		07
☑	8	0.0	1.053	10	180.0	0	0	0		07
☑	9	0.0	1.053	11	180.0	0	0	0		06
☑	10	0.0	1.053	16	180.0	0	0	0		05
☑	11	0.0	1.053	20	180.0	0	0	0		04
☑	12	0.0	1.053	25	180.0	0	0	0		03
☑	13	0.0	1.053	30	180.0	0	0	0		02
☑	14	0.0	1.053	35	180.0	0	0	0		01
☑	15	0.0	1.053	40	180.0	0	0	0		01
☑	16	0.0	1.053	50	180.0	0	0	0		02
☑	17	0.0	1.053	100	180.0	0	0	0		03

Figure 10 Environmental loads, for analyzed load cases.

6.2 Analysis results and verification

6.2.1 Total horizontal wave excitation force and phase

Figure 11 and Figure 12 provides the total horizontal wave excitation force and Figure 13 provides the phase, calculated by AquaSim using the "MacCamy-Fuchs" load formulation and the "Numerical diffraction" load formulation (NEMOH), but otherwise using the exact same analysis model, compared with the analytical long wave approximation, i.e. using Morison equation with added mass coefficient $C_a = 1.0$ and no drag ($C_D = 0.0$).

Very similar results are obtained between the "MacCamy-Fuchs" load formulation and the "Numerical diffraction" load formulation for all wave periods, as expected.

Furthermore, it is observed that both load formulations converge towards the analytical long wave approximation (Morison) for longer wave periods and good agreement is observed for wave periods $T \geq 6.0s$, which corresponds to wavelengths of $\lambda > 5 * D$, meaning the structure does not disturb the waves significantly and diffraction is of less importance, as expected.

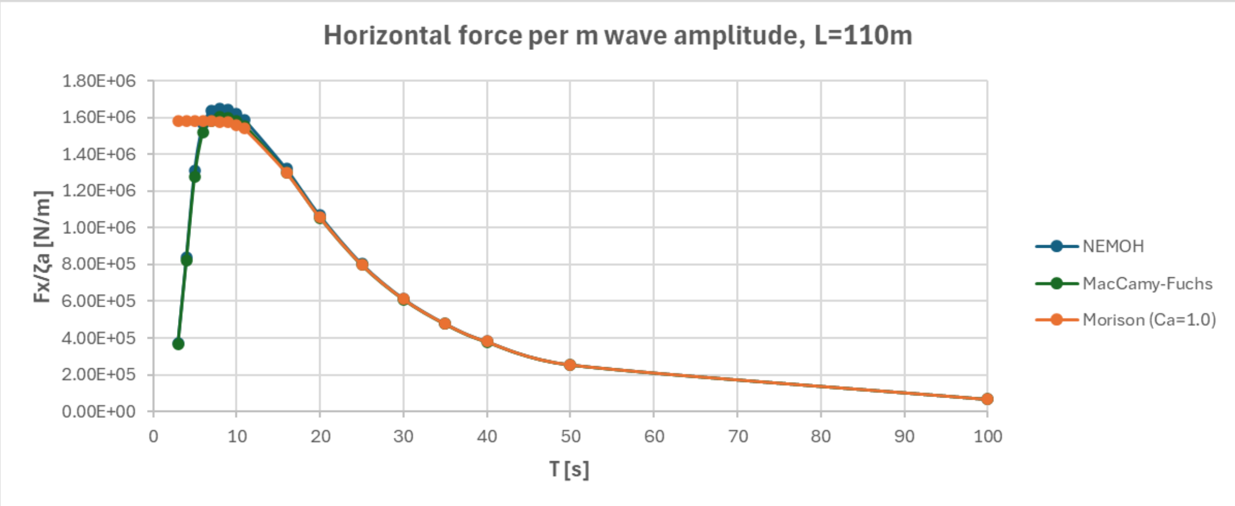

Figure 11 Comparison of horizontal force per m wave amplitude.

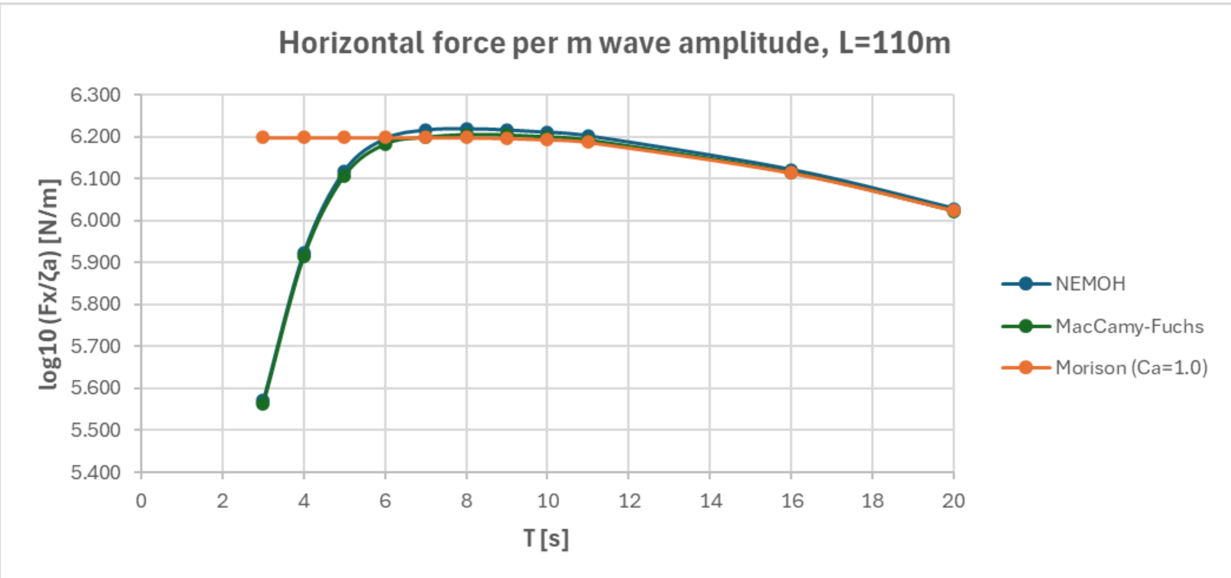

Figure 12 Comparison of horizontal force per m wave amplitude, with log10 scale and cut-off at wave period T = 20s.

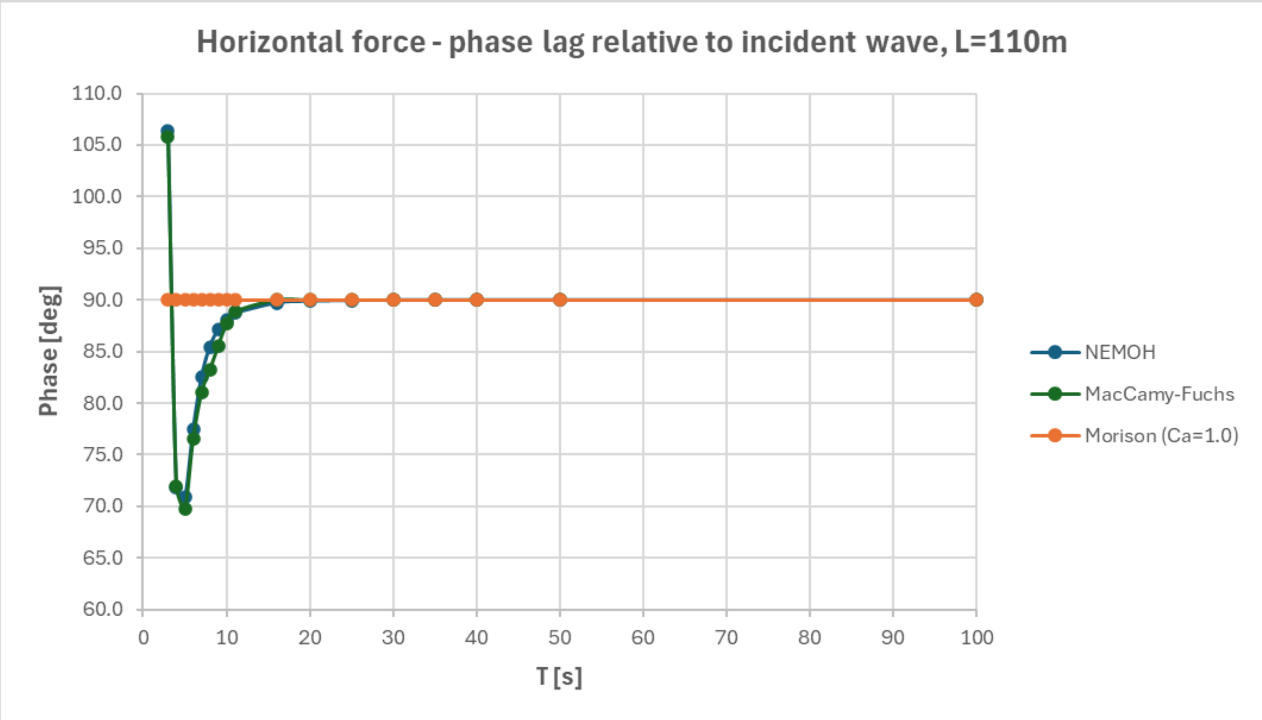

Figure 13 Comparison of the horizontal force phase lag relative to incident wave.

6.2.2 Added mass and hydrodynamic damping

Figure 14 provides the added mass and hydrodynamic damping acting on each panel as calculated by AquaSim using the “MacCamy-Fuchs” load formulation, while Table 3 provides a comparison between the total added mass and total hydrodynamic damping of the structure calculated by AquaSim and by analytical calculations.

Both the values of the added mass and hydrodynamic damping acting on each panel and the total for the entire structure obtained using the “MacCamy-Fuchs” load formulation are as expected and compare well to the analytical calculations.

Table 3 Comparison of total added mass and total hydrodynamic damping calvulated by AquaSim and by analytical calculations.

Parameter	**AquaSim**	**Analytical**	**Difference [-]**
A11 [tonne]	8796.2	8796.2	1.00
B11 [Ns/m]	8796230.0	8796235.7	1.00
A22 [tonne]	8796.2	8796.2	1.00
B22 [Ns/m]	8796230.0	8796235.7	1.00
A33 [tonne]	8796.2	8796.2	1.00
B33 [Ns/m]	8796230.0	8796235.7	1.00

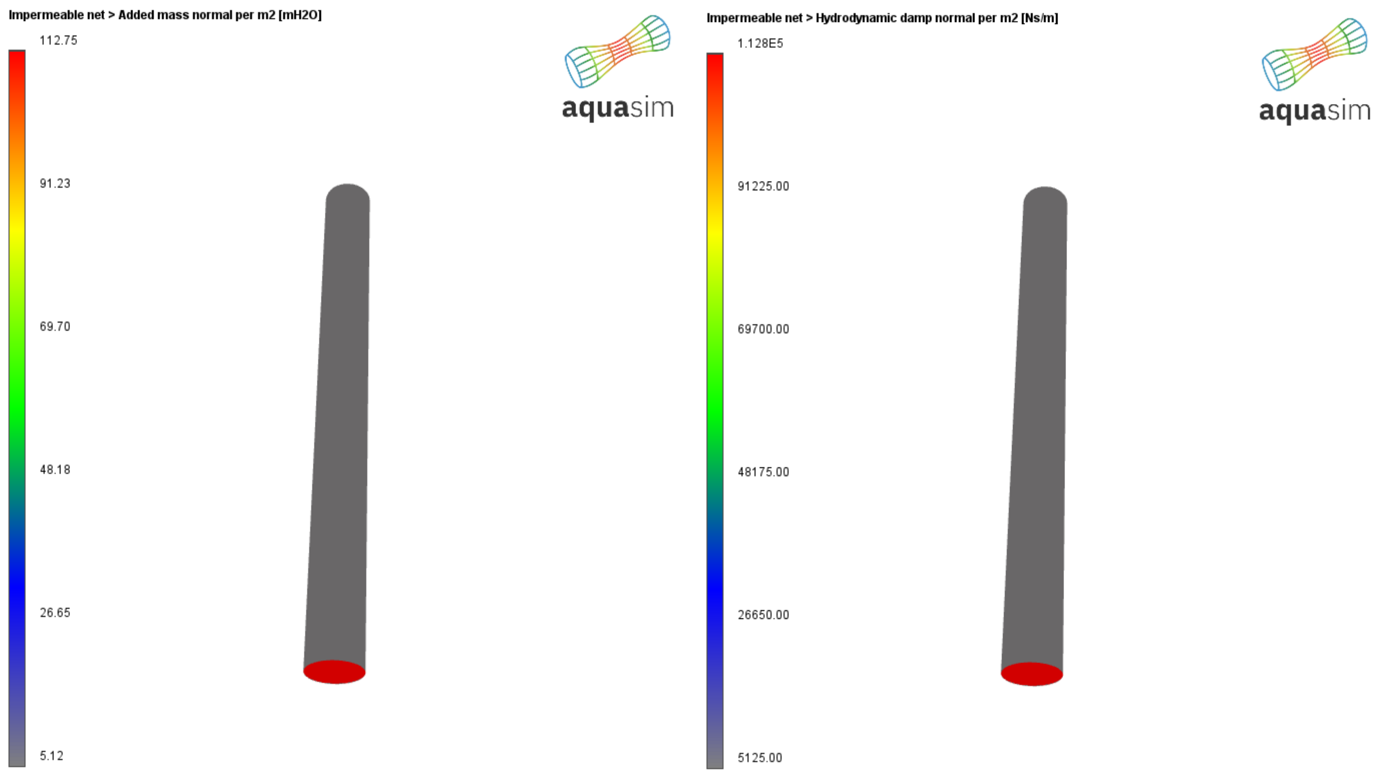

Figure 14 Added mass (left) and hydrodynamic damping (right), calculated by AquaSim using the "MacCamy-Fuchs" load formulation.

7 Notes on additional aspects

For additional details regarding preprocessing, meshing, drift forces, added mass waterline corrections, finite water depth, other aspects with respect to irregular waves, current-wave interactions, arbitrary motions/deformations, negative values for added mass and hydrodynamic damping, results/postprocessing ("avz-file", "PFAT-file" and "Hydro-file") and more, see tutorial (Aquastructures AS, 2025a).

8 Summary

In this tutorial we have seen how we can use the "MacCamy-Fuchs" load formulation for hydrodynamic analysis in AquaSim. Furthermore, we have described the theoretical background, assumptions and validity of this load formulation. Calculation and verification of the added mass and hydrodynamic damping obtained using this load formulation has also been covered.

Lastly a case study was performed, for a vertical circular cylinder with fixed radius and finite length, comparing the total horizontal wave excitation force and corresponding phase, calculated by AquaSim using the "MacCamy-Fuchs" load formulation, the "Numerical diffraction" load formulation and calculations using the analytical long wave approximation (Morison equation with $C_a = 1.0$ and $C_D = 0.0$). The results were as expected and showed good agreement.

9 References

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10 Revision comments

Revision no.	**Date**	**Comment**
1.0	20.10.2025	First publication

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