Drag coefficient for net panels

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Summary:
This document describes an revised method for calculation of drag on nets, that is offered in AquaSim, compared to what has previously been the established method (Berstad, Walaunet, & Heimstad, 2012).

1	14.06.2024	AJB	ISH & HNM	Drag coefficient nets revised, "Energy method" implemented
Revision no.	Date	Author	Verified by	Description

Content

1 Introduction

This document presents the default drag coefficient used in AquaSim for permeable net panels.

2 Theory

2.1 Net geometry

Consider a mesh shown in Figure 1.

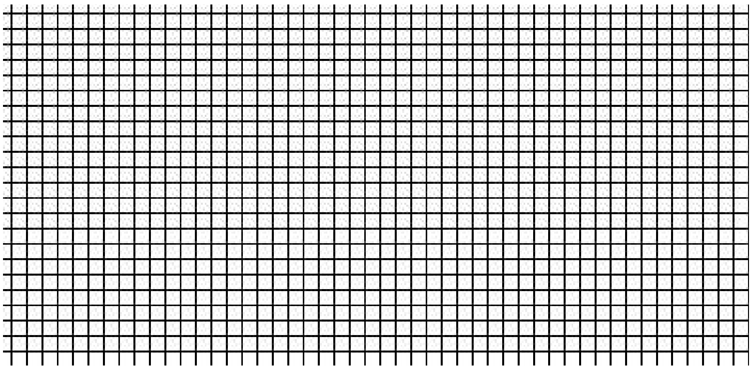


Figure 1 Mesh alternatively seen as a "screen"

If we zoom into the net seen in Figure 1, it looks like seen in Figure 2 where local definitions are introduced.

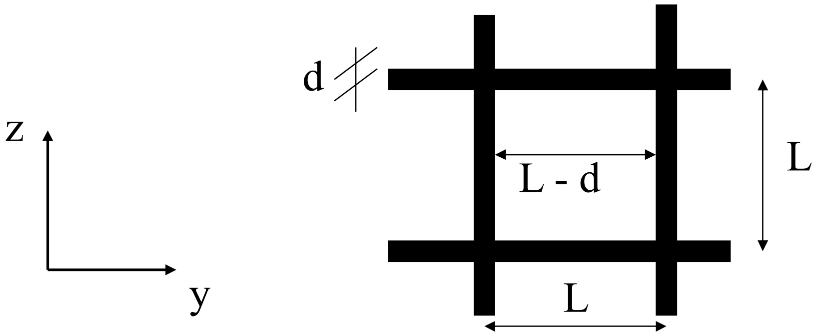


Figure 2 Definition to L and d of net twines

Figure 2 is from (Berstad, Walaunet, & Heimstad, 2012) and

- L is the distance, center-center between adjacent twines in both directions.
- d is the diameter of each twine in both directions.

2.2 Solidity

The most common formal definition to Solidity (Sn) is $Sn = {A_e}/{A}$, where A_e is the area casting shadow from a light perpendicular to the net and A is the total area of the net. For an ideal knotless mesh as shown in Figure 2 a mathematical expression for *Sn* can be formulated as:

$$Sn_m = \frac{2d}{L} - \frac{d^2}{L^2}$$

Historically meshes were made with knots. This leads to higher solidity due to extra net material at knots. A term having been used by e.g. (Løland, 1991) is:

$$Sn_{kn} = \frac{2d}{L} + \frac{kd^2}{4L^2}$$

where k is a constant, typically 1 or 2. Another simplified definition is found as:

$$Sn_{2D} = \frac{2d}{L}$$

Equation 1

This is often denoted the "2D solidity" since it basically is based on summing diameters in both directions. This can be a good balance since most nets are not mathematically perfect with an example seen in Figure 3.

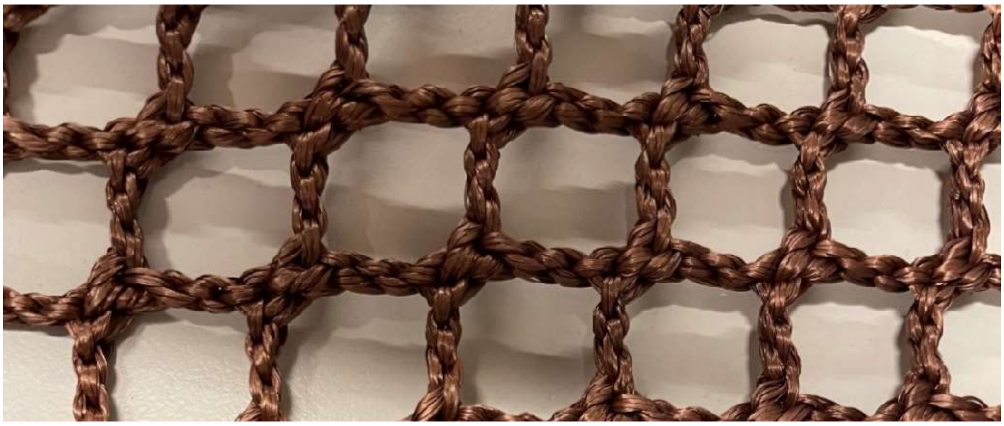


Figure 3 Net example

2.3 Drag forces on net membranes

Compare the mesh in Figure 2 by considering it as a baseline and mesh around as seen in Figure 4.

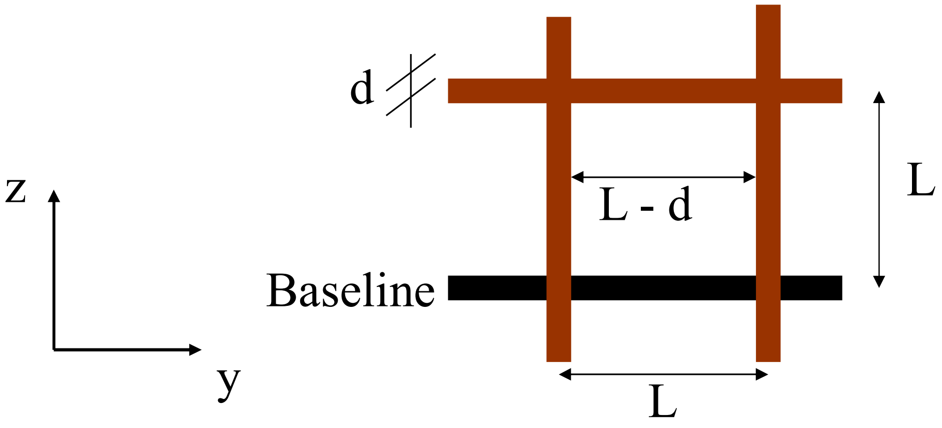


Figure 4 One twine denoted as baseline (from Berstad et. al. 2012)

2.3.1 The AquaSim 2012 drag coefficient

In Berstad et al (2012) an assessment is made by considering the difference between water flowing through a single twine (black in Figure 4) vs flowing through a single twine, with additional obstacles as the brown twines in Figure 4 and it is shown how a relation between the drag for flow around a single twine and for flow through a net as seen in Figure 5 can be stated as:

$$Cd_{mem} = Cd_{cyl} \frac{1}{(1 - \frac{Sn}{2})^3}$$

Equation 2

Equation 2 is used as the default is AquaSim where Sn the 2D solidity Sn_{2D}.

- Cd_{cyl} is the drag coefficient for a single twine.
- Cd_{mem} is the drag coefficient for the net relative to the total area of twines

Consider a net with area A and a flow at velocity *u* penetrating the net perpendicular as shown in Figure 5.

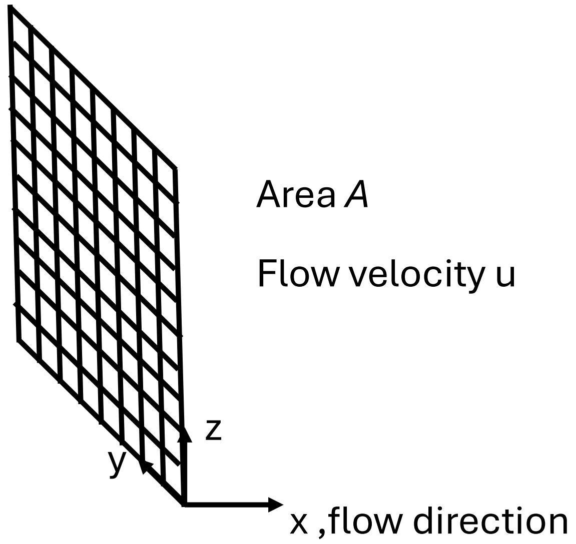


Figure 5 Screen with water flowing through having a cross section areal *A* to the flow.

Expressing the solidity in Equation 2 as a function of the total area of the screen (i.e. membrane element), A , the drag coefficient will be

$$Cd_{ms} = Cd_{mem}Sn = Cd_{cyl}\frac{Sn}{(1-\frac{Sn}{2})^3}$$

Equation 3

For cases where Sn is above 17%, the default Cd_{cyl} in AquaSim is 1. This means

$$Cd_{ms} = \frac{Sn}{(1-\frac{Sn}{2})^3}$$

Equation 4

The drag force F_{drag} for flow perpendicular to a net with area A is then

$$F_{drag} = \frac{1}{2}\rho Cd_{ms}Au^2$$

Equation 5

Introducing Equation 3 into Equation 5, the following expression is derived:

$$F_{drag} = \frac{\rho A Sn}{2(1-\frac{Sn}{2})^3}u^2$$

Equation 6

u is the flow velocity into the net seen in Figure 6. In AquaSim version 2.19.0 and earlier it has been assumed that $u = v$, see Figure 6, however the revised drag coefficient presented in Chapter 2.4 aims to account for the fact that $u \neq v$, as will be elaborated further in more detail in the next section.

2.4 Enhanced drag coefficient for net panels

In Berstad et al (2012) there is no considerations of how to find the flow velocity u relative to the undisturbed water flow velocity *v*. In the analysis, the drag coefficient in Equation 6 is applied to the undisturbed flow velocity, *v*. The objective of this section is to enhance that approach by finding the relation between the flow velocity *u* at the mesh and the undisturbed flow velocity *v*. This is approached by following the *1D* approach in e.g. Hansen (2008). 1D approach means that flow is symmetric abouts a centre line in the direction of the flow illustrated in Figure 6.

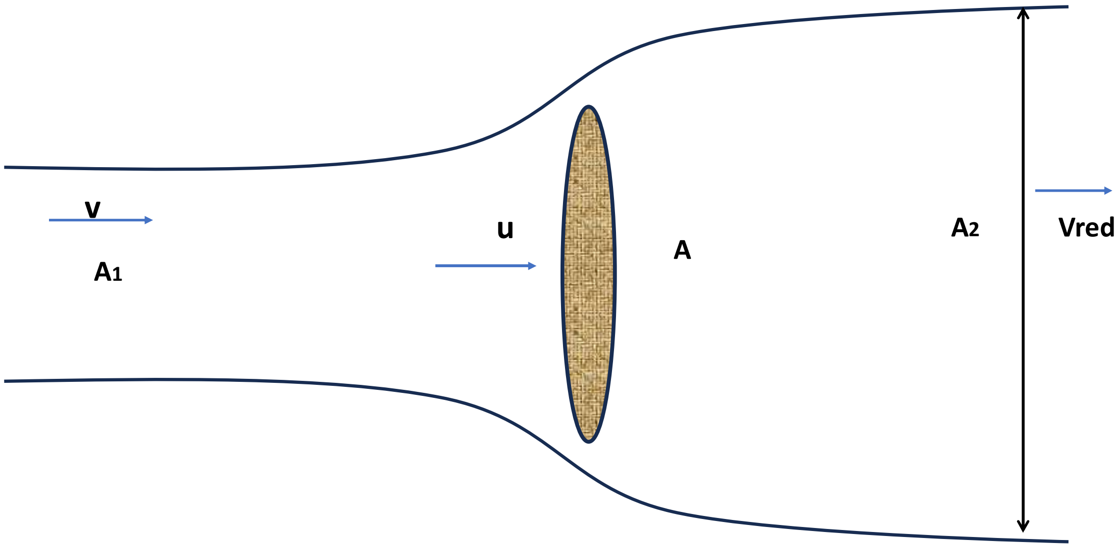


Figure 6 Flow through disc symmetric about the central axis.

This means that the net is assumed to be a circular disc as seen in Figure 6. By definition, the drag coefficient express the relation between F_{drag} and an undisturbed inflow velocity, *v* as given in Equation 7.

$$F_{drag} = \frac{1}{2}\rho CdAv^2$$

Equation 7

Meaning

$$Cd = \frac{2F_{drag}}{A\rho v^2}$$

Equation 8

Inserting Fdrag from Equation 6 into Equation 8 gives

$$Cd = \frac{Sn}{(1 - \frac{Sn}{2})^3}\frac{u^2}{v^2}$$

Equation 9

From Equation 9 it is seen that if the drag coefficient in Equation 4 shall be applicable as drag coefficient relative to the incident flow velocity, *v*, it must be adjusted for the difference in velocity between *u* and *v* squared.

In order for find the difference in velocity between *u* and *v* we follow Hansen (2008) whom introduces an "induction factor" *a* as

$$u = v(1 - a)$$

Equation 10

Meaning

$$\frac{u}{v} = (1 - a)$$

Equation 11

and

$$\frac{u^2}{v^2} = (1 - a)^2$$

Equation 12

By this definition of *a*, Equation 9 can be expressed as

$$Cd = \frac{Sn}{(1 - \frac{Sn}{2})^3}(1 - a)^2$$

Equation 13

Hansen (2008) consider rotating rotor blades which for each element in the BEM method. Equation 6.23 in Hansen (2008) is:

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma C_N} + 1}$$

Equation 14

Equation 14 is in the vocabulary of Hansen (2008). By the Hansen (2008) definitions,

- $\sigma = Sn$
- $C_N = Cd_{cyl}$
- $Sin^2\phi = 1$

This means that for the net considered in Figure 6 *a* can be expressed as

$$a = \frac{1}{\frac{4}{SnCd_{cyl}} + 1}$$

Equation 15

Using $Cd_{cyl} = 1$ which is the basis in AquaSim this means *a* can be expressed as,

$$a = \frac{1}{\frac{4}{Sn} + 1}$$

Equation 16

Or as

$$a = \frac{Sn}{4 + Sn}$$

Equation 17

This means

$$1 - a = 1 - \frac{Sn}{4 + Sn}$$

Equation 18

And

$$(1 - a)^2 = \left(1 - \frac{Sn}{4 + Sn}\right)^2$$

Equation 19

Now Cd can be found as

$$Cd = \frac{Sn}{(1 - \frac{Sn}{2})^3}\left(1 - \frac{Sn}{4 + Sn}\right)^2$$

Equation 20

According to Hansen (2008), Fig. 6.5 the expression in Equation 16 is valid for $a<0.4$. However, as seen from Equation 17, $a = 0.2$, corresponds to a solidity of $Sn=1.0$, and the validity in this context therefore extends to $a = 0.2$, since it is not relevant to consider $Sn>1.0$. The validity of Equation 16 with respect to *a* is therefore not the limitation for the present application.

Making a Taylor series expansion at Sn = 0 with the two first terms gives

$$\left(1 - \frac{Sn}{4 + Sn}\right)^2 \approx 1 - \frac{1}{2}Sn$$

Equation 21

Such that a simplified expression for *Cd* can be expressed as

$$Cd = \frac{Sn}{(1 - \frac{Sn}{2})^2}$$

Equation 22

Note that the simplification in Equation 22 is nonconservative such that in AquaSim, Equation 20 is used directly.

In AquaSim does also have an opportunity for Cd_{cyl} of another value than 1.0. In this case Equation 20 will be

$$Cd = \frac{Cd_{cyl}Sn}{(1-\frac{Sn}{2})^3}\left(1 - \frac{Cd_{cyl}Sn}{4 + Cd_{cyl}Sn}\right)^2$$

Equation 23

2.5 Comparison to empirical data

Figure 7 shows drag as function of solidity.

- Føre_2022 is the expression deducted by the polynomial $Cd = 1.872Sn^2 + 1.057Sn - 0.053$ presented in Føre et al (2022) equation 10. The paper states that this polynominal fit is applicable for cd [0.18-0.36] so the line is limited to this range.
- AquaSim_2012: Is the default Cd from AquaSim 2012, Equation 4.
- AquaSim_2024 is Equation 20.
- Føre U = 1 m/s refers to Føre et al (2022) Fig. 9
- Føre Rn 2000 refers to Føre et al (2022) Fig. 9
- Føre Rn 3000 refers to Føre et al (2022) Fig. 9

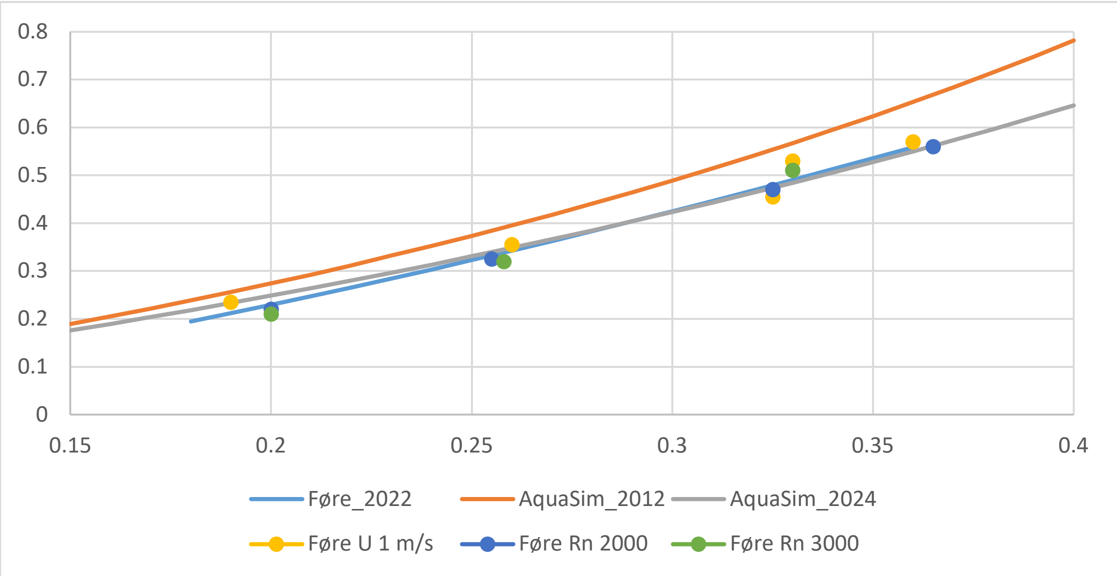


Figure 7 Drag coefficients as function of solidity.

As seen from Figure 7 the Equation 20 fits very well with both empirical data and the polynomial fit in Føre et al (2022).

The empirical data in Føre et al (2022) is based on finding the solidity by the photographic method which is more precise and usually estimate 10-15% higher solidity than combining diameter and solidity formulae. Hence using the 2024 version of the AquaSim drag coefficient for nets means one should input solidity based on photography or increase solidity 10-15 %.

2.6 AquaSim implementation validation

A 2x2 net has been established in AquaSim as shown in Figure 8 and Figure 9

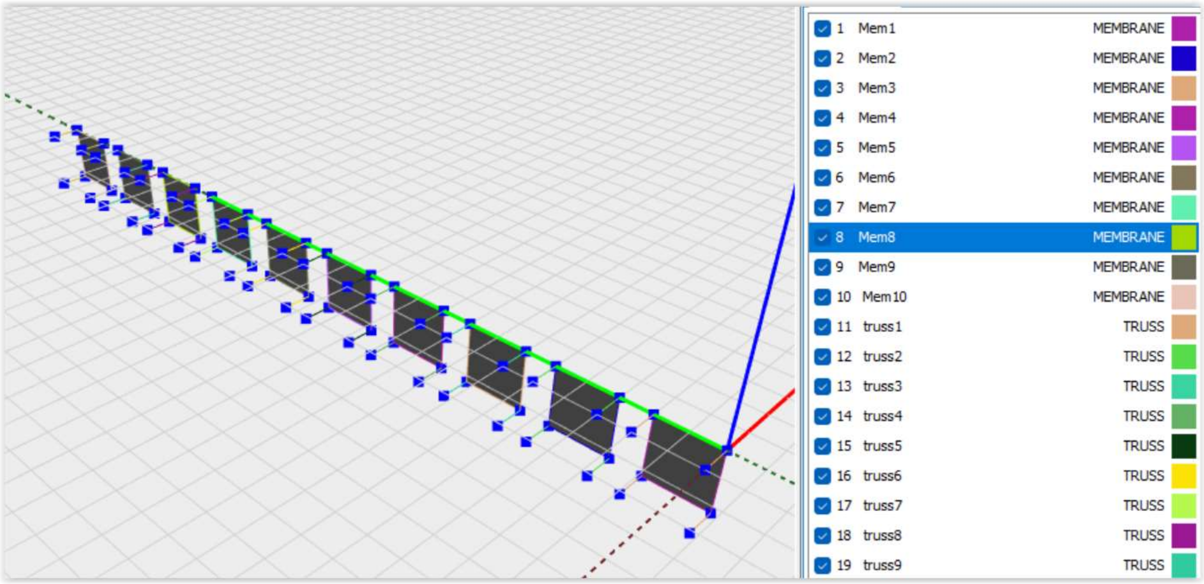


Figure 8 Analysis model with varying solidity on nets.

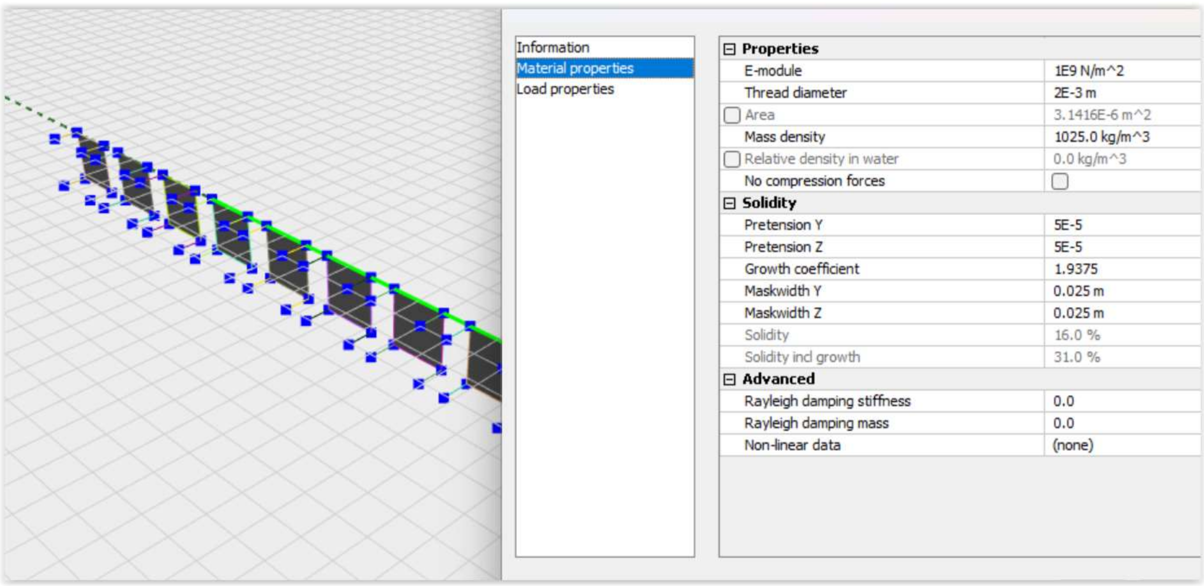


Figure 9 Varying growth coefficient per net.

Trusses were modelled in the corners are shown in Figure 10 and fixed in one end and free to move in the current direction at the connection to the nets.

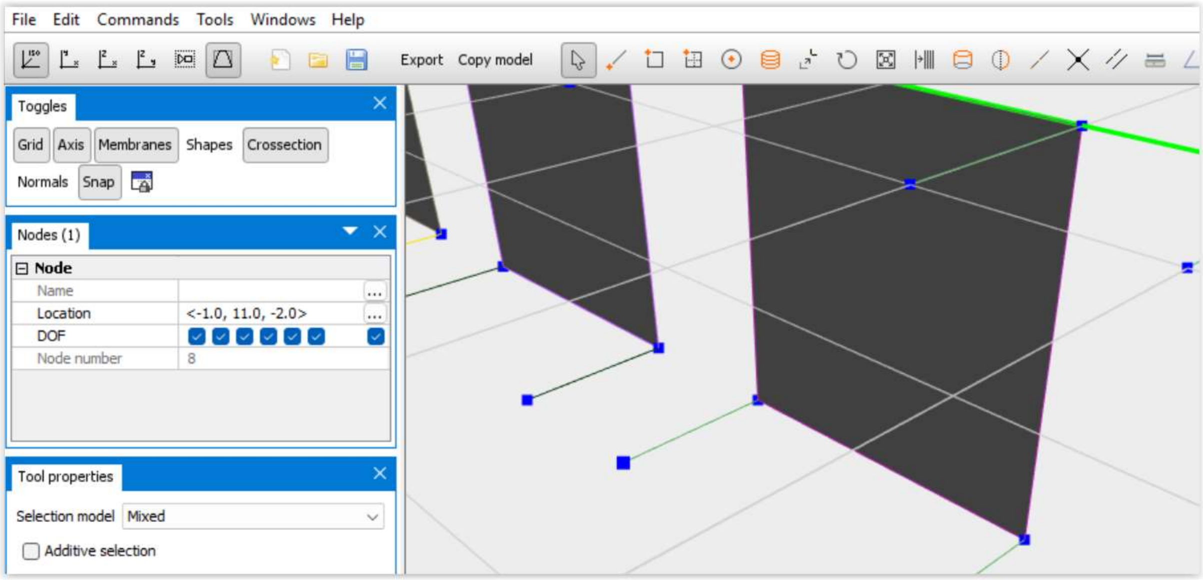
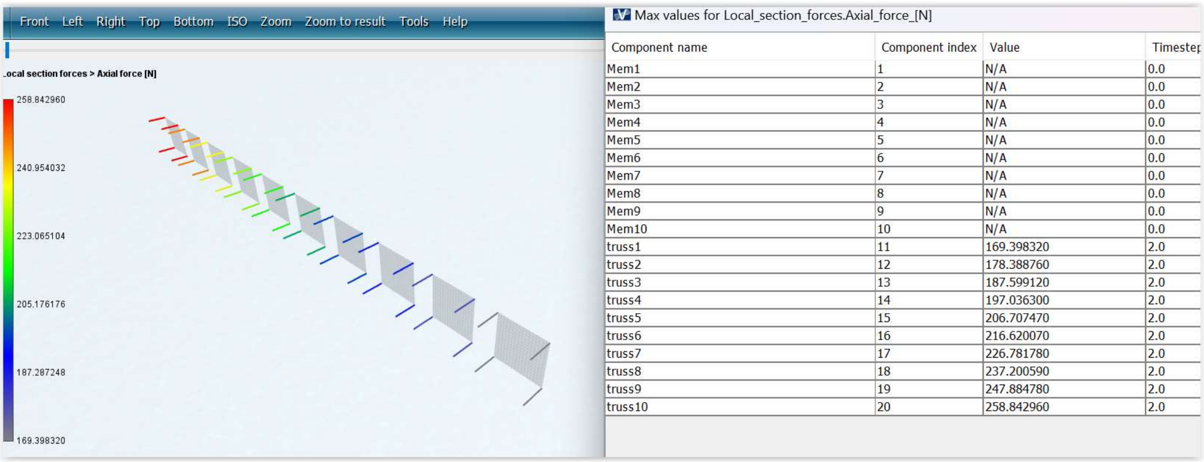


Figure 10 Trusses in corners.

Forces were found in the trusses seen in Figure 10 as shown in Figure 11.



Component name	Component index	Value	Timestep
Mem1	1	N/A	0.0
Mem2	2	N/A	0.0
Mem3	3	N/A	0.0
Mem4	4	N/A	0.0
Mem5	5	N/A	0.0
Mem6	6	N/A	0.0
Mem7	7	N/A	0.0
Mem8	8	N/A	0.0
Mem9	9	N/A	0.0
Mem10	10	N/A	0.0
truss1	11	169.398320	2.0
truss2	12	178.388760	2.0
truss3	13	187.599120	2.0
truss4	14	197.036300	2.0
truss5	15	206.707470	2.0
truss6	16	216.620070	2.0
truss7	17	226.781780	2.0
truss8	18	237.200590	2.0
truss9	19	247.884780	2.0
truss10	20	258.842960	2.0

Figure 11 Analysis results.

Forces derived in the AquaSim analysis were used to derive the corresponding drag coefficient Figure 12 shows comparison, in Figure 12:

- Analytic 2024 is the drag coefficient calculated from Equation 20.
- AquaSim 2012 are results calculated from the Formulation introduced to AquaSim in 2012 (Berstad et al 2012), Equation 4.
- AquaSim 2024 are the drag coefficients extracted from the forces in the AquaSim 2024 analysis which shall comply with Analytic 2024.
- 2024 Solidity +10% are forces calculated from AquaSim 2024 analysis (drag coefficient calculated from Equation 20) but with solidity increased 10%. This is recommended if using AquaSim 2024 for analysis where solidity has been determined

by calipers or other means known to underestimate solidity compared to the photographic technique.

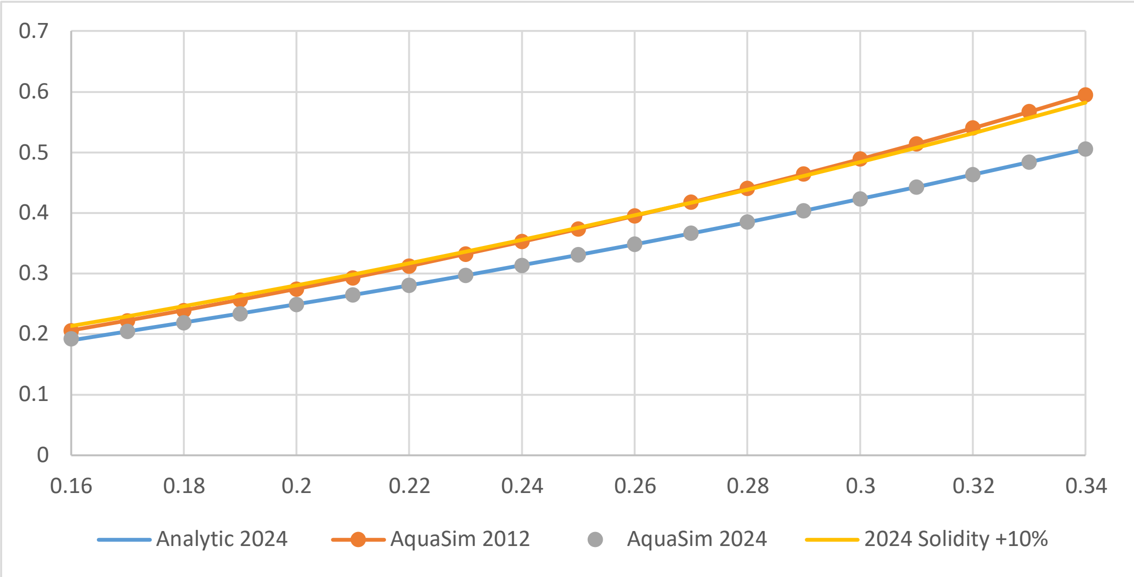


Figure 12 Comparison analysis methods

As seen from Figure 12 results compare well.

3 References

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