Added mass, hydrodynamic damping and convolution integral in AquaSim

TN-FOU-101830-4

Revision 1

Report no.:	TR-FOU-100004-7		
Date of this revision:	30.10.2025		
Number of pages:	13		
Distribution:	Open		
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Summary:

The option to account for convolution-based hydrodynamic forces on Beam and Membrane components has been introduced to AquaSim. This implementation allows for including memory-effects that arise from frequency-dependent added mass and damping in time-domain simulations.

A test case demonstrates that including the convolution term can lead to a reduction in structural response and mooring loads under irregular sea states. Consequently, analyses performed without convolution may provide more conservative estimates.

1	30.10.2025	AJB	ISH	First revision
Revision no.	Date	Author	Verified by	Description

Content

1 Introduction

This document outlines the options available in AquaSim for analysing hydrodynamic forces. When the added mass is frequency dependent, meaning it varies with the wave period, the system response cannot be fully captured using constant coefficients.

AquaSim provides two methods for accounting for this frequency dependence. By default, the added mass and hydrodynamic damping is calculated at the mean zero-crossing period of the wave spectrum. However, as the added mass depends on the frequency it introduces a memory-effect, leading to the appearance of a convolution integral in the time-domain equations of motion.

This convolution integral, often referred to as the retardation function, represents the cumulative influence of past fluid-structure interactions on the current motion. In AquaSim, this effect can be included as an alternative to using constant values at the mean zero-crossing period. The methodology to compute this contribution is outlined in this document.

2 Theoretical background

The conventional approach for modeling radiation forces in time-domain is governed by Cummins' equation, which incorporates a convolution integral to represent the memory effect of radiated waves (Cummins, 1962). The convolution integral depends on a retardation function derived from frequency-dependent radiation damping, capturing how past velocities affect the current force. However, direct numerical evaluation of this integral can be computationally intensive and prone to convergence issues, particularly in nonlinear systems subjected to irregular wave spectra. These challenges stem from the need to store and integrate over a long velocity history, which can amplify numerical errors, especially when high-frequency components are present in the wave spectrum.

To address these issues, a modified approach has been developed that splits the radiation force into two components: an explicit damping term, based on a characteristic wave frequency, and a modified convolution integral that accounts for the residual damping. This document outlines the theoretical foundation of this modified approach, presents the necessary mathematical formulations. Further test cases are presented and discussed.

2.1 Frequency-Domain Representation of Radiation forces

In the frequency domain, the radiation force exerted on a floating structure due to its oscillatory motion at frequency ω is characterized by complex hydrodynamic coefficients: the added mass A(ω) and the radiation damping B(ω). Assuming harmonic motion $x(t) = \mathrm{R}\{\hat{x}e^{i\omega t}\}$, the corresponding radiation force can be expressed as:

$$F_{\mathrm{rad}}(\omega) = -i\omega[A(\omega) + iB(\omega)]\hat{x} = -\omega^2 A(\omega)\hat{x} - i\omega B(\omega)\hat{x}.$$

This relation captures how the structure's velocity and acceleration induce hydrodynamic forces due to radiated waves. While frequency-domain models are useful for linear analyses, time-domain simulations require a formulation that accommodates arbitrary motion histories and nonlinear effects.

2.2 Time-Domain Formulation: Cummins' Equation

To enable time-domain analysis, (Cummins, 1962) derived an equation of motion based on the system's impulse response. The radiation force is represented via a convolution integral that captures memory effects due to wave radiation:

$$(M + A_\infty)\ddot{x}(t) + \int_0^t K(t-\tau)\dot{x}(\tau)d\tau + Cx(t) = F_{\text{exc}}(t) + F_{\text{other}}(t),$$

where:

- M: Structural mass.
- A_∞: Added mass at infinite frequency.
- $\ddot{x}(t)$,$\dot{x}(t)$, x(t): Acceleration, velocity, and displacement.
- K(t): Retardation function (radiation kernel).
- C: Hydrostatic stiffness.
- $F_{exc}(t)$: Wave excitation force.
- $F_{other}(t)$: Other forces, such as nonlinear or viscous contributions

The radiation force, $F_{\text{rad}}(t)$, given by the convolution integral $\int_0^t K(t-\tau)\dot{x}(\tau)d\tau$, represents the memory effect of waves radiated by the structure's motion. The retardation function K(t) is derived from the frequency-dependent radiation damping B(ω) via the inverse Fourier transform. The retardation function K(t) captures the fluid's memory effect, transforming frequency- dependent damping into the time domain:

$$K(t) = \frac{2}{\pi}\sum_{i=1}^{N} c_{h(\omega_i)cos(\omega_i t)\Delta\omega_i}$$

where:

- •$ch(\omega_i)$: Hydrodynamic damping coefficient at frequency ω_i (kg/s),
- •ω_i: Wave frequency (rad/s),
- •$\Delta\omega_i$: Frequency increment (rad/s),
- •N: Number of frequency components.

This is an approximation of the inverse Fourier transform of $c_h(\omega)$.

2.3 The modified Convolution Approach

To improve numerical stability, the radiation force is reformulated by separating the radiation damping into an explicit term and a modified convolution integral:

$$F_{\mathrm{rad}}(t) = -B\dot{x}(t) - \int_0^t K'(t-\tau)\dot{x}(\tau)d$$

where:

- B = B(ω_z): Radiation damping evaluated at the characteristic frequency $\omega_z = 2\pi/Tz$, where Tz is the zero-crossing period of the wave spectrum (e.g., JONSWAP or Pierson-Moskowitz).
- K′(t): Modified retardation function, accounting for the residual damping B′(ω) = B(ω)− B.

This can be expressed in relation to the original retardation function:

$$K'(t) = K(t) - B\delta(t)$$

where δ(t) is the Dirac delta function, representing the constant damping B in the frequency domain. In numerical implementation, the delta function contribution is handled by the explicit damping term $-B\dot{x}(t)$, and the convolution integral evaluates the effect of *K′(t).*

The explicit damping term $-B\dot{x}(t)$ is computed using the current velocity *x(t),* making it a local force that does not require historical data. The modified convolution integral,$\int_0^t K'(t-\tau)\dot{x}(\tau)$, captures the dynamic memory effect but with a reduced amplitude. K′(t) is approximated by the inverse cosine transform of the residual damping spectrum:

$$K'(t) \approx \sum_{n=1}^{N} B'_n \cos(\omega_n t)$$

as $B'(\omega) = B(\omega) - B$ typically has a lower magnitude than *B(ω).*

2.4 Modified Radiation Force as a Sum over Wave Components

In the analysis, the irregular sea is described with N wave components, each characterized by an angular frequency ω_n and an associated spectral weight. In this context, the radiation force is expressed as a sum over these components, where the damping at each frequency contributes to the force via a corresponding oscillatory kernel. We define the total radiation force as:

$$F_{\mathrm{rad}}(t) = -B\,\dot{x}(t) - \sum_{n=1}^{N} B'_n \int_0^t \cos[\omega_n(t-\tau)]\,\dot{x}(\tau)\ d\tau$$

Or

$$F_{\text{rad}}(t_k) \approx -B\,\dot{x}(t_k) - \sum_{n=1}^{N} B'_n \sum_{j=0}^{k} \cos\left[\omega_n\left(t_k - t_j\right)\right] \dot{x}\left(t_j\right)\,\Delta t$$

with:

- $B = B(\omega_z)$ being the radiation damping evaluated at the characteristic wave mean crossing frequency $\omega_z = 2\pi/Tz$,
- $B'_n = B(\omega_n) - B$ as the residual damping for each component,
- ω_n as the discrete angular frequencies representing the sea state,
- N as the number of wave components used in the spectral description.
- $t_k = k\Delta t$ and $\{\dot{x}\}(t_j)$ is the velocity history at discrete time steps.

This spectral approach offers both physical transparency and numerical robustness. The convolution is reduced to a weighted sum of elementary cosine functions, each modulated by the velocity history $\dot{x}(\tau)$. The numerical implementation involves storing the velocity history and computing a convolution sum over past time steps for each wave component. While this is computationally more demanding than using a single damping term, so it is a balance as whether to apply this instead of values at the mean zero crossing period.

2.5 Pros and cons of including convolution integral

The pros of not including convolution are as follows:

- Simplicity: Easy to implement, requiring only a single set of hydrodynamic coefficients.
- Computational Efficiency: Avoids complex integrals, reducing simulation time.
- Suitability for Narrow-Banded Waves: Effective when the wave spectrum is dominated by a single frequency.

Whereas the downsides are:

- Reduced Accuracy: Neglects frequency-dependent variations, leading to errors in broadband wave conditions.
- No Memory Effects: Fails to account for the influence of past motions on current forces.
- Limited Applicability: Less suitable for irregular sea or systems with strong frequency dependence.

The advantages of including convolution are:

- High Accuracy: Captures frequency-dependent effects and memory effects, ideal for broadband waves.
- Physical Realism: Accounts for the influence of past motions, critical for systems with strong damping variations.
- Versatility: Suitable for nonlinear, transient, and coupled responses in irregular seas.

Whereas the downsides of this is:

- Computational Complexity: The convolution integral increases simulation time and resource demands.
- Numerical Challenges: Requires careful discretization to avoid instability or aliasing.

– Data Requirements: Needs comprehensive frequency-domain hydrodynamic data

A practical analysis strategy can be to perform the primary (bulk) analysis using coefficients at mean zero-crossing period, then evaluate the most critical or dynamically sensitive conditions through convolution integral.

3 Implementation to AquaSim

Hydrodynamic damping through convolution is only applicable for irregular wave conditions. For regular waves, AquaSim calculates added mass and damping at the corresponding wave period.

3.1 Choice of methodology in AquaEdit

In irregular waves, the default is to omit the convolution integral and apply values for added mass and hydrodynamic damping at the mean zero crossing period. If convolution is to be introduced on Beam components, it must be ticked off in the “Advanced” properties section as shown in Figure 1.

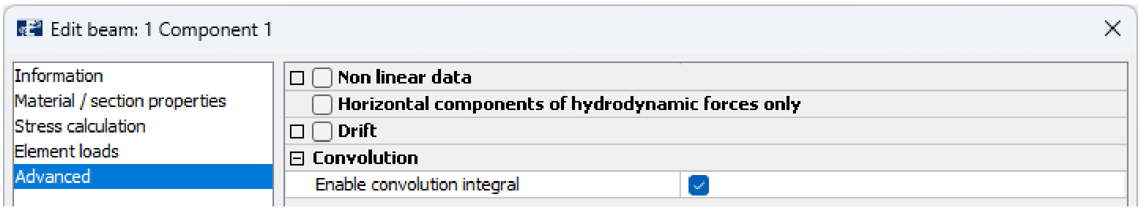

Figure 1 How to enable hydrodynamic damping by convolution for Beam with load formulation Hydrodynamic

For membrane panels where added mass and hydrodynamic damping is evaluated with numerical method, convolution integral is introduced in the section “Impermeable properties”, as seen in Figure 2.

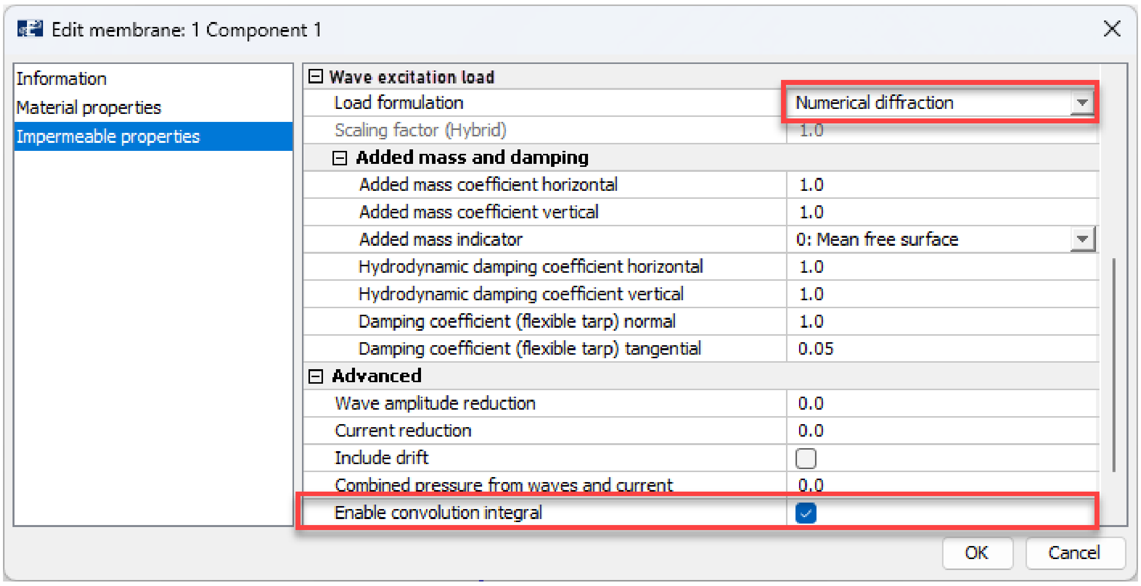

Figure 2 Enabling convolution for membrane panels

3.2 Evaluation of results

For Beam- and Membrane components that are exposed to hydrodynamic loads, results can be evaluated both as forces per component per timestep, and graphically in the result-files.

3.2.1 Convolution forces on membrane panels

Results are also written to file *#convolution.txt* where the convolution force to each component for each step of the analysis is presented as shown in Figure 3.

```
Convolution forces [N]
Step, Component,  Force X ,          Force Y,           Force Z
    1     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    2     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    3     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    4     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    5     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    6     1        0.00000000E+00     0.00000000E+00     0.00000000E+00
    7     1        0.17012026E+00     0.00000000E+00     0.00000000E+00
    8     1        0.53664047E-01     0.00000000E+00     0.00000000E+00
    9     1       -0.13459893E+00     0.00000000E+00     0.00000000E+00
   10     1       -0.45807867E+00     0.00000000E+00     0.00000000E+00
   11     1       -0.54586320E+00     0.00000000E+00     0.00000000E+00
   12     1       -0.40918115E+00     0.00000000E+00     0.00000000E+00
   13     1       -0.11320909E+00     0.00000000E+00     0.00000000E+00
   14     1        0.26642120E+00     0.00000000E+00     0.00000000E+00
   15     1        0.28407604E+00     0.00000000E+00     0.00000000E+00
   16     1        0.27564904E+00     0.00000000E+00     0.00000000E+00
```

Figure 3 Convolution force to each component for each timestep in the analysis

Where the convolution force F_{conv} is expressed as:

$$F_{\text{conv}}(t_k) = \sum_{n=1}^{N} B'_n \sum_{j=0}^{k} \cos[\omega_n(t_k - t_j)]\, \dot{x}(t_j)\ \Delta t$$

This force term represents the memory effect associated with the frequency-dependent added mass and is subtracted from the other external forces acting on the structure. The convolution force can be evaluated as a distributed load, expressed in terms of force per projected area, and resolved into its directional components along global coordinates (*x-, y-, z-*), as illustrated in Figure 4.

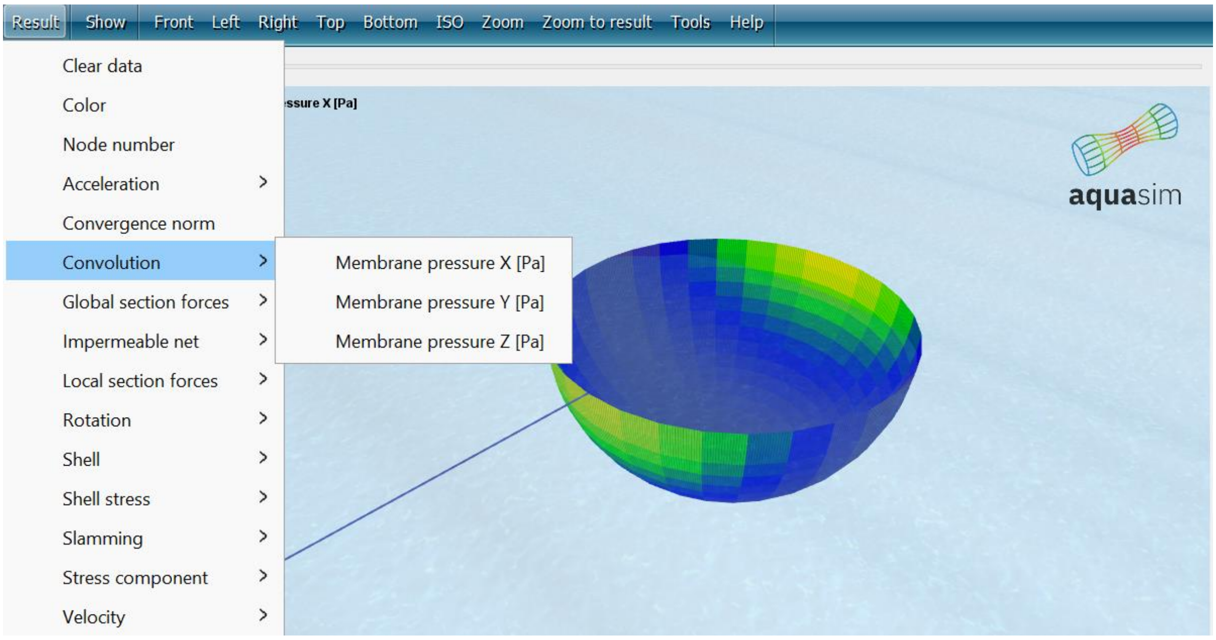

Figure 4 Convolution force presented as force per area in AquaView

3.2.2 Convolution forces on beams

Convolution forces on beams are written to file in the same way as for membranes and results are shown in terms of force per m beam in the graphic result (avz) file as shown in Figure 5.

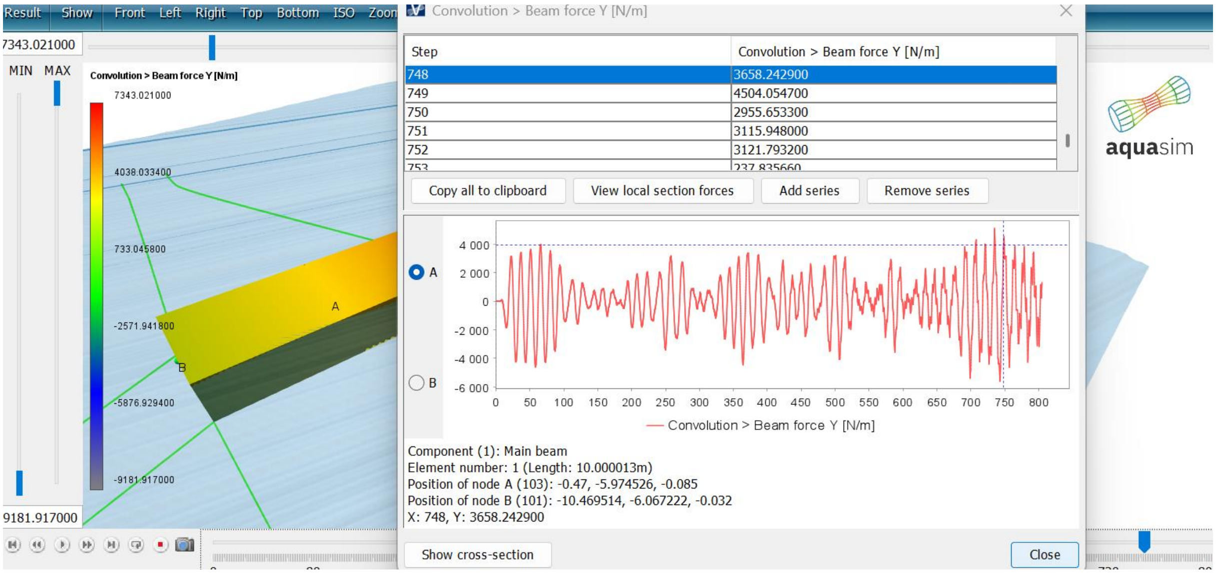

Figure 5 Convolution force presented as force per meter in AquaView

Figure 6 shows max axial forces in mooring lines at a time instant for the analyzed irregular sea state.

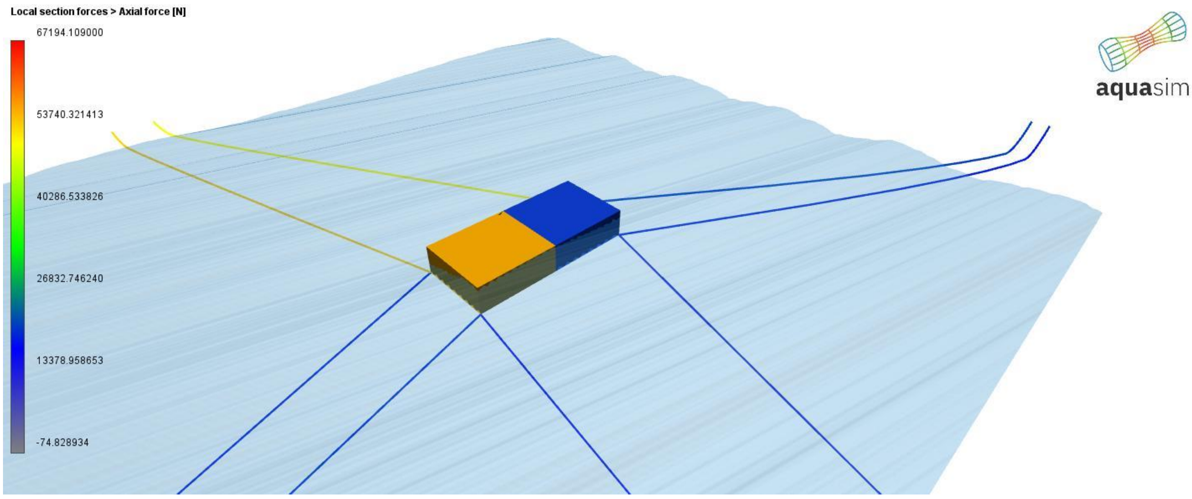

Figure 6 Axial forces in mooring lines

Table 1 shows max forces in the eight mooring lines from the analysis, and how much the maximum forces are reduced in case convolution is included.

Table 1 Comparison max forces with- and without convolution

	Max force [kN]	Max force [kN]	Convolution
	With convolution	No convolution	Reduction [%]
Mooring line 1 - 52mm	19.84	20.27	2.11 %
Mooring line 2 - 52mm	18.84	18.84	0.00 %
Mooring line 3 - 52mm	18.84	18.84	0.00 %
Mooring line 4 - 52mm	22.16	23.04	3.81 %
Mooring line 5 - 52mm	23.58	23.80	0.89 %
Mooring line 6 - 52mm	55.88	58.88	5.10 %
Mooring line 7 - 52mm	55.91	58.83	4.96 %
Mooring line 8 - 52mm	20.65	20.70	0.27 %

It is seen from Table 1 that forces are reduced by up to 5% for mooring line no. 6. In this table, all forces include pretension. Further, Table 2 present the same comparison only with pretension and response from current subtracted.

Table 2 Maximum forces where pretension and response from current subtracted

	Max force [kN]	Max force [kN]	Convolution
	With convolution	No convolution	Reduction [%]
Mooring line 1 - 52mm	1.00	1.43	29.94 %
Mooring line 2 - 52mm	0.00	0.00	-
Mooring line 3 - 52mm	0.00	0.00	-
Mooring line 4 - 52mm	2.66	3.54	24.78 %
Mooring line 5 - 52mm	3.90	4.12	5.14 %
Mooring line 6 - 52mm	14.72	17.72	16.93 %
Mooring line 7 - 52mm	14.76	17.68	16.52 %
Mooring line 8 - 52mm	1.81	1.86	2.95 %

As shown in Table 2, which only considers wave forces, the resulting additional mooring forces due to wave excitation are reduced by up to approximately 30%. This is a significant effect. Furthermore, it is observed that incorporating the convolution term generally leads to a reduction of forces in the mooring lines. This indicates that using the classical formulation without convolution is a conservative estimate.

4 Conclusion

The option to account for convolution-based hydrodynamic forces on Beam and Membrane components has been introduced to AquaSim. This implementation allows for including memory-effects that arise from frequency-dependent added mass and damping in time-domain simulations.

A test case demonstrates that including the convolution term can lead to a reduction in structural response and mooring loads under irregular sea states. Consequently, analyses performed without convolution may provide more conservative estimates.

5 Refernces

Cummins, W. E. (1962). *The Impulse Response Function and Ship Motions.* Schiffstechnick, vol. 9, pp.101-109.