aquastructures

Technical note

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Revision 1

Added mass, hydrodynamic damping and convolution integral in AquaSim

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Summary:

The option to introduce forces caused by convolution for hydrodynamic response has been introduced to AquaSim both for beam elements with hydrodynamic loads and membrane elements.

A test case shows this may reduce response in irregular seas.

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1 Introduction

This document describes the options relevant when doing analysis hydrodynamic forces on AquaSim. When the added mass is frequency-dependent, varying with the wave period, the system's response cannot be fully captured using simple constant coefficients.

AquaSim offers two ways to account to this. As default, added mass and hydrodynamic damping is calculated at the mean zero crossing period of the spectructum. As the added mass is frequency-dependent it introduces a memory effect, leading to the appearance of a convolution integral in the time domain equations of motion. This convolution integral, often referred to as the retardation function, represents the cumulative influence of past fluid-structure interactions on the current motion. This can be included in the AquaSim analysis as an alternative to using values at the mean zero crossing period. How this is calculated is outlined in this document.

2 Theoretical background

The standard approach to modeling radiation forces in the time domain is based on Cummins' equation, which incorporates a convolution integral to represent the memory effect of radiated waves [1]. The convolution integral depends on a retardation function derived from frequency-dependent radiation damping, capturing how past velocities affect the current force. However, direct numerical evaluation of this integral can be computationally intensive and prone to convergence issues, particularly in nonlinear systems subjected to irregular wave spectra. These challenges stem from the need to store and integrate over a long velocity history, which can amplify numerical errors, especially when high-frequency components are present in the wave spectrum.

To address these issues, a modified approach has been developed that splits the radiation force into two components: an explicit damping term, based on a characteristic wave frequency, and a modified convolution integral that accounts for the residual damping. This document outlines the theoretical foundation of this modified approach, presents the necessary mathematical formulations. Further test cases is presented and discussed.

2.1 Frequency-Domain Representation of Radiation Forces

In the frequency domain, the radiation force exerted on a floating structure due to its oscillatory motion at frequency ω is characterized by complex hydrodynamic coefficients: the added mass A(ω) and the radiation damping B(ω). Assuming harmonic motion $x(t) = \mathrm{R}\{\hat{x}e^{i\omega t}\}$, the corresponding radiation force can be expressed as:

$$F_{\mathrm{rad}}(\omega) = -i\omega[A(\omega) + iB(\omega)]\hat{x} = -\omega^2 A(\omega)\hat{x} - i\omega B(\omega)\hat{x}.$$

This relation captures how the structure's velocity and acceleration induce hydrodynamic forces due to radiated waves. While frequency-domain models are useful for linear analyses, time-domain simulations require a formulation that accommodates arbitrary motion histories and nonlinear effects.

2.2 Time-Domain Formulation: Cummins' Equation

To enable time-domain analysis, Cummins [1] derived an equation of motion based on the system's impulse response. The radiation force is represented via a convolution integral that captures memory effects due to wave radiation:

$$(M + A_\infty)\ddot{x}(t) + \int_0^t K(t-\tau)\dot{x}(\tau)d\tau + Cx(t) = F_{\text{exc}}(t) + F_{\text{other}}(t),$$

where:

- M : Structural mass.
- A_∞: Added mass at infinite frequency.
- $\ddot{x}$(t),$\dot{x}$(t), x(t): Acceleration, velocity, and displacement.
- K(t): Retardation function (radiation kernel).
- C: Hydrostatic stiffness.
- F_{exc}(t): Wave excitation force.
- F_{other}(t): Other forces, such as nonlinear or viscous contributions

The radiation force, $F_{\text{rad}}(t)$, given by the convolution integral $\int_0^t K(t-\tau)\dot{x}(\tau)d\tau$, represents the memory effect of waves radiated by the structure's motion. The retardation function K(t) is derived from the frequency-dependent radiation damping B(ω) via the inverse Fourier transform The retardation function K(t) captures the fluid's memory effect, transforming frequency- dependent damping into the time domain:

$$K(t) = \frac{2}{\pi}\sum_{i=1}^{N} c_{h(\omega_i) cos(\omega_i t) \Delta\omega_i}$$

where:

- •$c_h(\omega_i)$: Hydrodynamic damping coefficient at frequency ω_i (kg/s),
- •ω_i: Wave frequency (rad/s),
- •$\Delta\omega_i$: Frequency increment (rad/s),
- •N : Number of frequency components.

This is an approximation of the inverse Fourier transform of c_h(ω).

2.3 The modified Convolution Approach

To improve numerical stability, the radiation force is reformulated by separating the radiation damping into an explicit term and a modified convolution integral:

$$F_{\text{rad}}(t) = -B\dot{x}(t) - \int_0^t K'(t-\tau)\dot{x}(\tau)d$$

where:

- B = B(ω_z): Radiation damping evaluated at the characteristic frequency ω_z = 2π/Tz , where Tz is the zero-crossing period of the wave spectrum (e.g., JONSWAP or Pierson-Moskowitz).
- K′(t): Modified retardation function, accounting for the residual damping B′(ω) = B(ω)− B.

This can be expressed in relation to the original retardation function:

$$K'(t) = K(t) - B\delta(t)$$

where δ(t) is the Dirac delta function, representing the constant damping B in the frequency domain. In numerical implementation, the delta function contribution is handled by the explicit damping term $-B\dot{x}(t)$,and the convolution integral evaluates the effect of *K′(t)*.

The explicit damping term $-B\dot{x}(t)$ is computed using the current velocity *x(t)*, making it a local force that does not require historical data. The modified convolution integral,$\int_0^t K'(t-\tau)\dot{x}(\tau)$, captures the dynamic memory effect but with a reduced amplitude. K′(t) is approximated by the inverse cosine transform of the residual damping spectrum:

$$K'(t) \approx \sum_{n=1}^{N} B'_n \cos(\omega_n t)$$

as $B'(\omega) = B(\omega) - B$ typically has a lower magnitude than *B(ω)*.

2.4 Modified Radiation Force as a Sum over Wave Components

In the analysis, the irregular sea is described with N wave components, each characterized by an angular frequency ωn and an associated spectral weight. In this context, the radiation force is expressed as a sum over these components, where the damping at each frequency contributes to the force via a corresponding oscillatory kernel. We define the total radiation force as:

$$F_{\text{rad}}(t) = -B\,\dot{x}(t) - \sum_{n=1}^{N} B'_n \int_0^t \cos[\omega_n(t-\tau)]\,\dot{x}(\tau)\;d\tau$$

Or

$$F_{\text{rad}}(t_k) \approx -B\,\dot{x}(t_k) - \sum_{n=1}^{N} B'_n \sum_{j=0}^{k} \cos\big[\omega_n\big(t_k - t_j\big)\big]\,\dot{x}\big(t_j\big)\;\Delta t$$

with:

- $B = B(\omega_z)$ being the radiation damping evaluated at the characteristic wave mean crossing frequency $\omega_z = 2\pi/Tz$,
- $B'_n = B(\omega_n) - B$ as the residual damping for each component,
- ω_n as the discrete angular frequencies representing the sea state,
- *N* as the number of wave components used in the spectral description.
- $t_k = k\Delta t$ and $\{\dot{x}\}(t_j)$ is the velocity history at discrete time steps.

This spectral approach offers both physical transparency and numerical robustness. The convolution is reduced to a weighted sum of elementary cosine functions, each modulated by the velocity history $\dot{x}(\tau)$. The numerical implementation involves storing the velocity history and computing a convolution sum over past time steps for each wave component. While this is computationally more demanding than using a single damping term, so it is a balance as weather to apply this instead of values at the mean zero crossing period.

2.5 Pros and cons of including convolution integral

The pros of not including convolution is as follows

- Simplicity: Easy to implement, requiring only a single set of hydrodynamic coefficients.
- Computational Efficiency: Avoids complex integrals, reducing simulation time.
- Suitability for Narrow-Banded Waves: Effective when the wave spectrum is dominated by a single frequency.

Whereas the downsides are:

- Reduced Accuracy: Neglects frequency-dependent variations, leading to errors in broadband wave conditions.
- No Memory Effects: Fails to account for the influence of past motions on current forces.
- Limited Applicability: Less suitable for irregular seas or systems with strong frequency dependence.

The advantages for including convolution are:

- High Accuracy: Captures frequency-dependent effects and memory effects, ideal for broadband waves.
- Physical Realism: Accounts for the influence of past motions, critical for systems With strong damping variations.
- Versatility: Suitable for nonlinear, transient, and coupled responses in irregular seas.

Where as the downsides of this is:

- Computational Complexity: The convolution integral increases simulation time and resource demands.
- Numerical Challenges: Requires careful discretization to avoid instability or aliasing.
- Data Requirements: Needs comprehensive frequency-domain hydrodynamic data

A practical approach may be to do the bulk analysis with using values at mean zero crossing period, then evaluate the most susceptible conditions by convolution.

3 Implementation to AquaSim.

Hydrodynamic damping by convolution is only relevant for irregular waves, for regular waves, added mass and hydrodynamic damping is evaluated to the wave period in AquaSim.

3.1 Choice of methodology in AquaEdit

In irregular waves, the default is to omit the convolution integral and us values for added mass and hydrodynamic damping at the mean zero crossing period. If convolution is to be introduced, it must be ticked off in the “Advanced” properties section as shown in Figure 1.

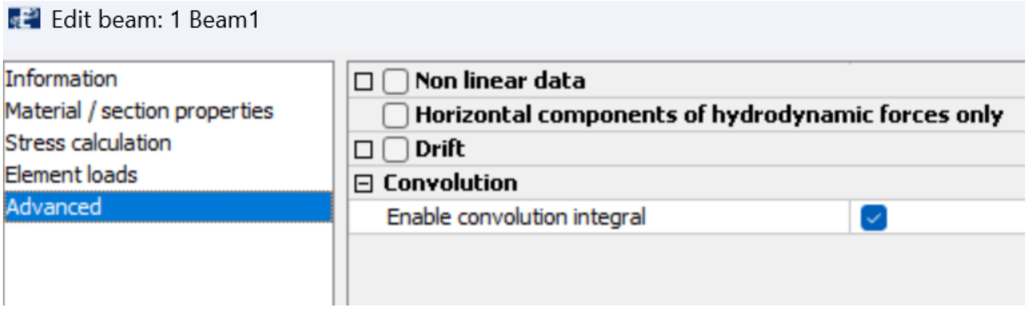

Figure 1 How to enable hydrodynamic damping by convolution

For panels where added mass and damping is evaluated numerically, it is introduced in the "Impermeable properties" section as shown in Figure 2. In case convolution integral shall be included, the "Enable convolution integral" box should be ticked.

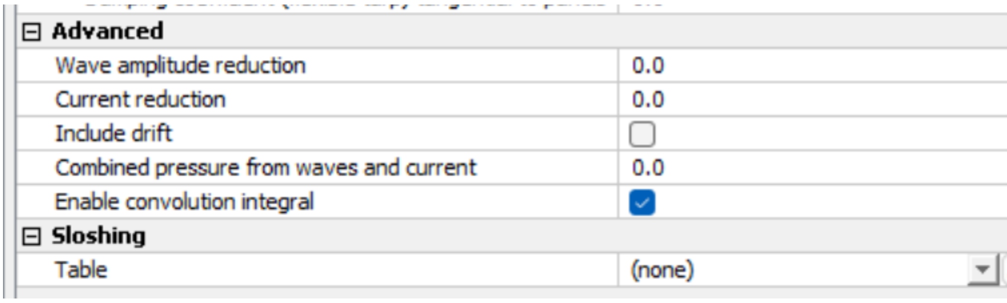

Figure 2 Enabling convolution for panels

3.2 Results evaluation

Both for beams and panels exposed to hydrodynamic loads, results can be evaluated both as forces per component per timestep and graphically in the results file.

3.2.1 Forces on membranes

Results are also written to file # convolution.txt where the convolution force to each component for each step of the analysis is presented as shown in Figure 3.

Convolution forces [N]

Step,	Component,	Force X ,	Force Y,	Force Z
1	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
2	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
3	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
4	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
5	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
6	1	0.00000000E+00	0.00000000E+00	0.00000000E+00
7	1	0.17012026E+00	0.00000000E+00	0.00000000E+00
8	1	0.53664047E-01	0.00000000E+00	0.00000000E+00
9	1	-0.13459893E+00	0.00000000E+00	0.00000000E+00
10	1	-0.45807867E+00	0.00000000E+00	0.00000000E+00
11	1	-0.54586320E+00	0.00000000E+00	0.00000000E+00
12	1	-0.40918115E+00	0.00000000E+00	0.00000000E+00
13	1	-0.11320909E+00	0.00000000E+00	0.00000000E+00
14	1	0.26642120E+00	0.00000000E+00	0.00000000E+00
15	1	0.28407604E+00	0.00000000E+00	0.00000000E+00
16	1	0.27564904E+00	0.00000000E+00	0.00000000E+00

Figure 3 Convolution force to each component for each step of the analysis

Where the convolution force , F_{conv} is

$$F_{\text{conv}}(t_k) = \sum_{n=1}^{N} B_n' \sum_{j=0}^{k} \cos[\omega_n(t_k - t_j)]\, \dot{x}(t_j)\, \Delta t$$

And is subtracted to other acting forces. The convolution force can be evaluated in terms for force per area projected and how its distributed in each direction (*x*-, *y*-, *z*-) as shown in Figure 4.

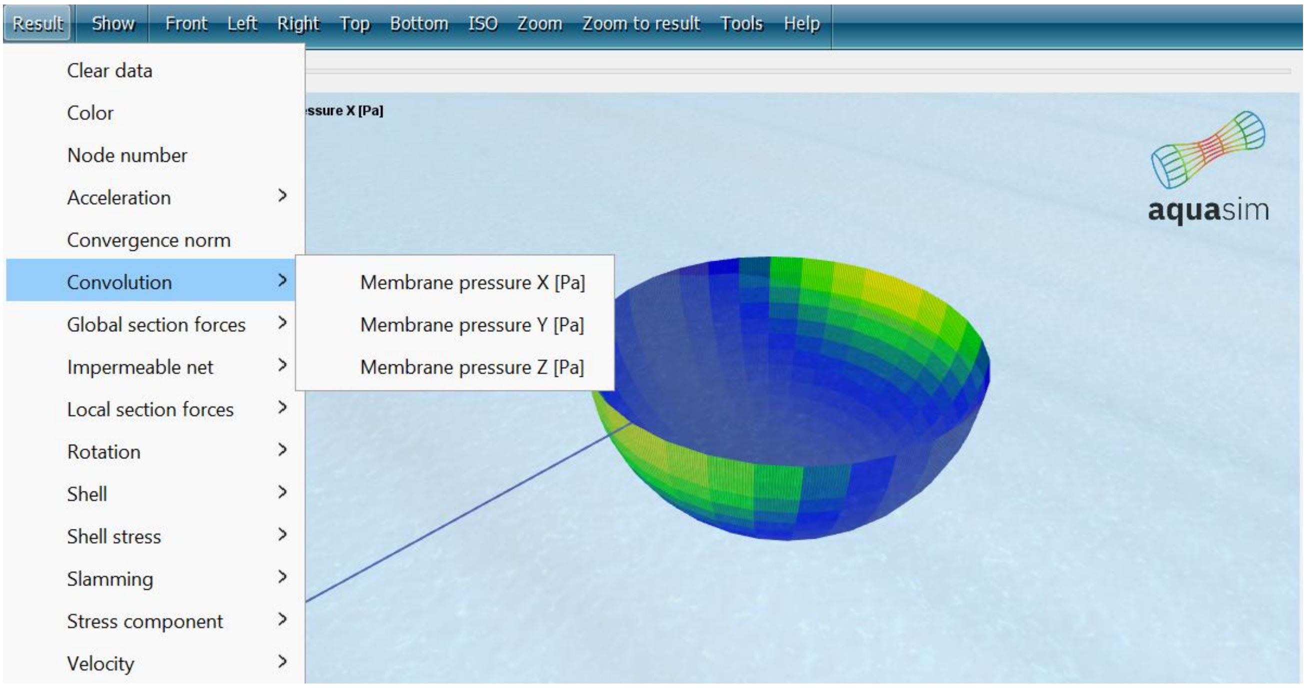

Figure 4 convolution

3.2.2 Convolution forces on beams

Convolution forces on beams are written to file in the same way as for membranes and results are shown in terms of force per m beam in the graphic result (avz) file as shown in Figure 5.

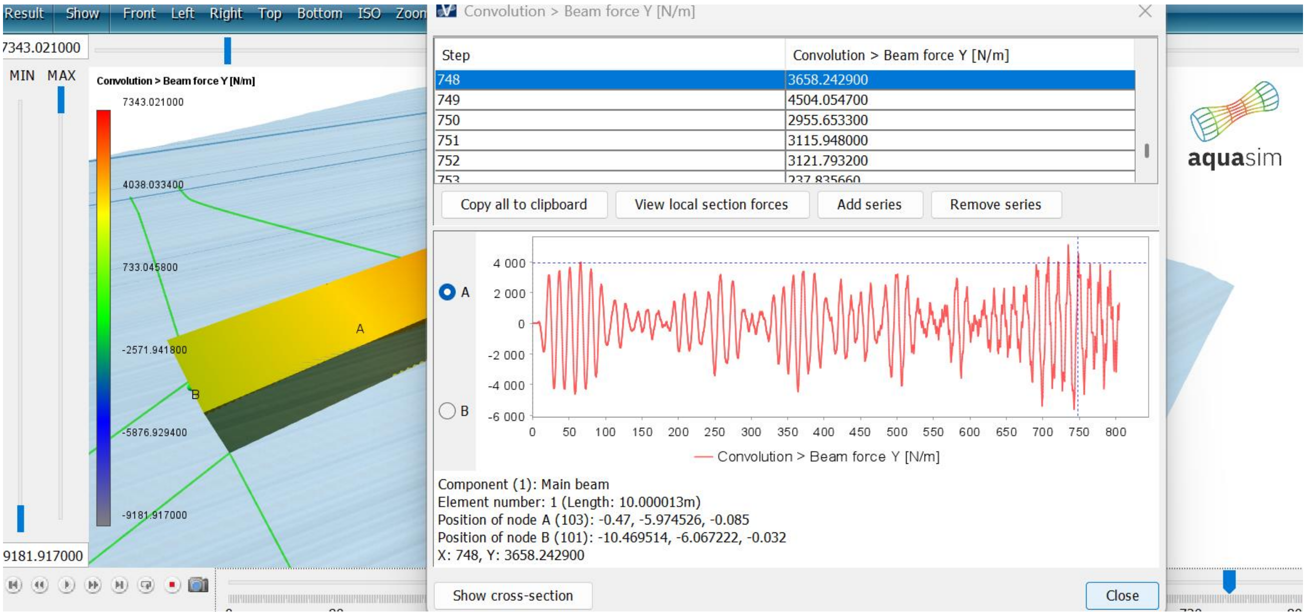

Figure 5 Convolution forces to beam.

Figure 6 shows axial forces in mooring lines at a time instant for the analysed irregular sea condition.

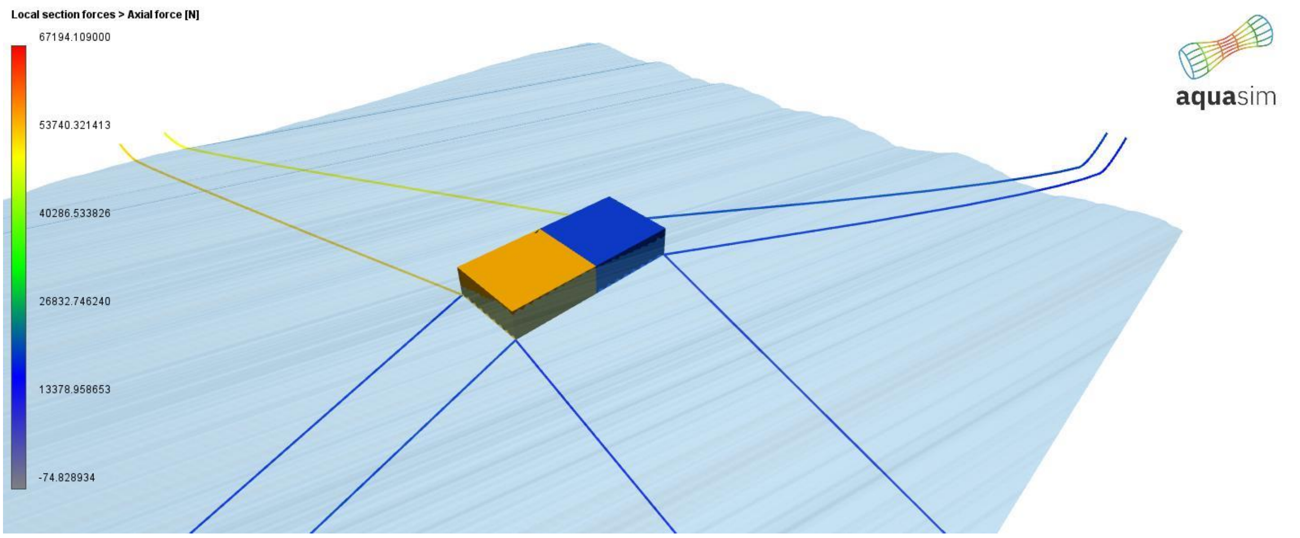

Figure 6 Axial forces in mooring lines

Table 1 shows max forces in mooring lines from the analysis and how much max forces are reduced in case convolution is accounted for.

Table 1

	Max force [kN]	Max force [kN]	Convolution
	With convolution	**No convolution**	**Reduction [%]**

Mooring line 1 - 52mm	19.84	20.27	2.11 %
Mooring line 2 - 52mm	18.84	18.84	0.00 %
Mooring line 3 - 52mm	18.84	18.84	0.00 %
Mooring line 4 - 52mm	22.16	23.04	3.81 %
Mooring line 5 - 52mm	23.58	23.80	0.89 %
Mooring line 6 - 52mm	55.88	58.88	5.10 %
Mooring line 7 - 52mm	55.91	58.83	4.96 %
Mooring line 8 - 52mm	20.65	20.70	0.27 %

As seen from Table 1 forces are reduced up to 5 % for case with high load. In Table 1 all mooring forces are included including pretension.

Table 2 shows the same comparison, but with pretension and response from current subtracted.

Table 2

	Max force [kN] With convolution	Max force [kN] No convolution	Convolution Reduction [%]
Mooring line 1 - 52mm	1.00	1.43	29.94 %
Mooring line 2 - 52mm	0.00	0.00	NA
Mooring line 3 - 52mm	0.00	0.00	NA
Mooring line 4 - 52mm	2.66	3.54	24.78 %
Mooring line 5 - 52mm	3.90	4.12	5.14 %
Mooring line 6 - 52mm	14.72	17.72	16.93 %
Mooring line 7 - 52mm	14.76	17.68	16.52 %
Mooring line 8 - 52mm	1.81	1.86	2.95 %

As seen from Table 2, considering wave forces only, there is a reduction in resulting extra mooring forces caused by waves of up to 30% which is of significance. It is also seen that accounting for convolution in general reduces mooring forces. This means that using the classic formulation without convolution in this case is conservative.

4 Conclusion

The option to introduce forces caused by convolution for hydrodynamic response has been introduced to AquaSim both for beam elements with hydrodynamic loads and membrane elements.

A test case shows this may reduce response in irregular seas.

5 References

[1] Cummins, W. E. (1962) “The Impulse Response Function and Ship Motions,” Schiffstechnik, vol. 9, pp. 101–109, 1962