# Cross sectional properties of beam in AquaEdit

## Theoretical formulation and validation

TR-FOU-100004-2

Revision 1

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Summary:

This document presents how AquaEdit calculates cross sectional properties of beams generated from the Wizard. The properties are validated against analytical solution. Results show good correspondence.


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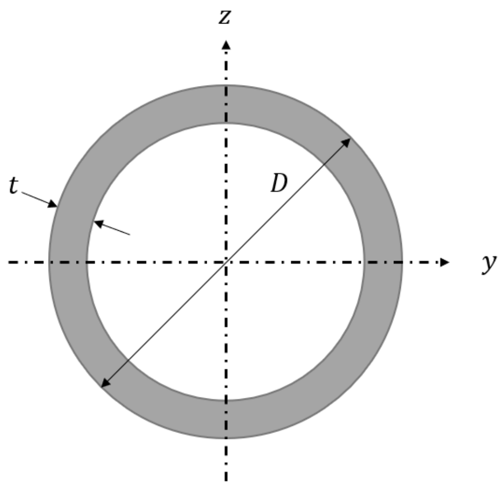
# Content

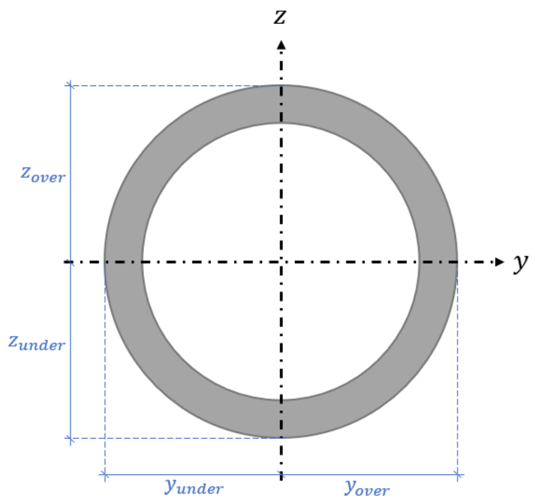
# 1 Introduction

This document describes how cross-sectional properties are calculated for cross sections generated from the Data source wizard beam elements in AquaEdit.

# 2 Theory

## 2.1 Tube



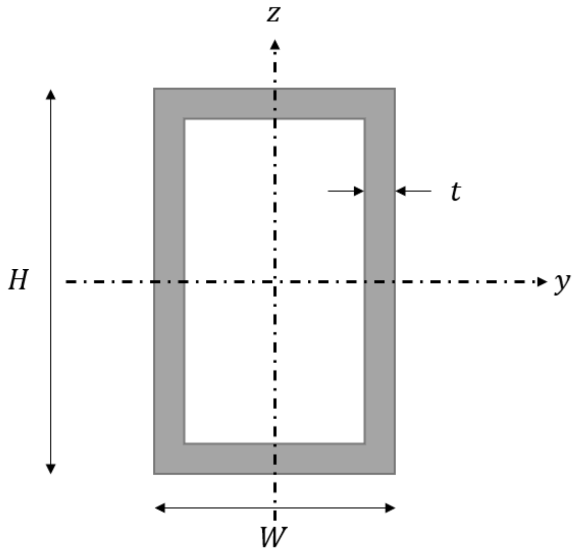



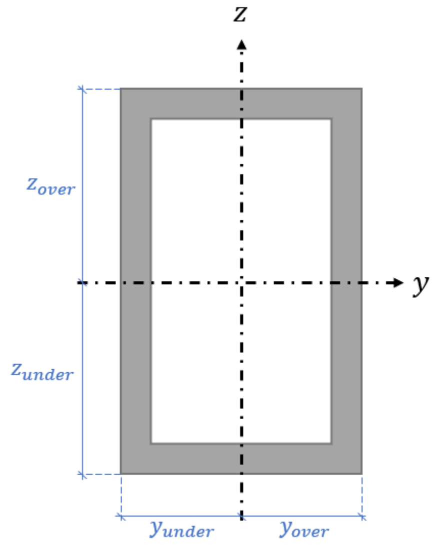

A tube is a circular cross section with an outer diameter $D$ and a thickness $t$.

| Description | Formula | Comment |
|---|---|---|
| **Inner diameter of tube** | $d = D - 2 \cdot t$ | |
| **Cross-sectional area** | $A = \pi \cdot \left( \left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2 \right)$ | |
| **Second moment of area due to bending** | $I_y = I_z = \frac{\pi}{64}(D^4 - d^4)$ | The second moment of area due to bending for a tube is equal in y- and z-direction due to axis symmetry. |
| **Second moment of area due to torsion** | $I_t = I_p = I_y + I_z = \frac{\pi}{32}(D^4 - d^4)$ | For circular cross sections, the second moment of area due to torsion $I_t$ will correspond to the polar area of moment $I_p$. See e.g., (Larsen, 1990) Ch. 5.2.2 |
| **Bending: distance to neutral axis** | $z_{over} = D/2$<br>$y_{over} = D/2$<br>$z_{under} = D/2$<br>$y_{under} = D/2$ | |

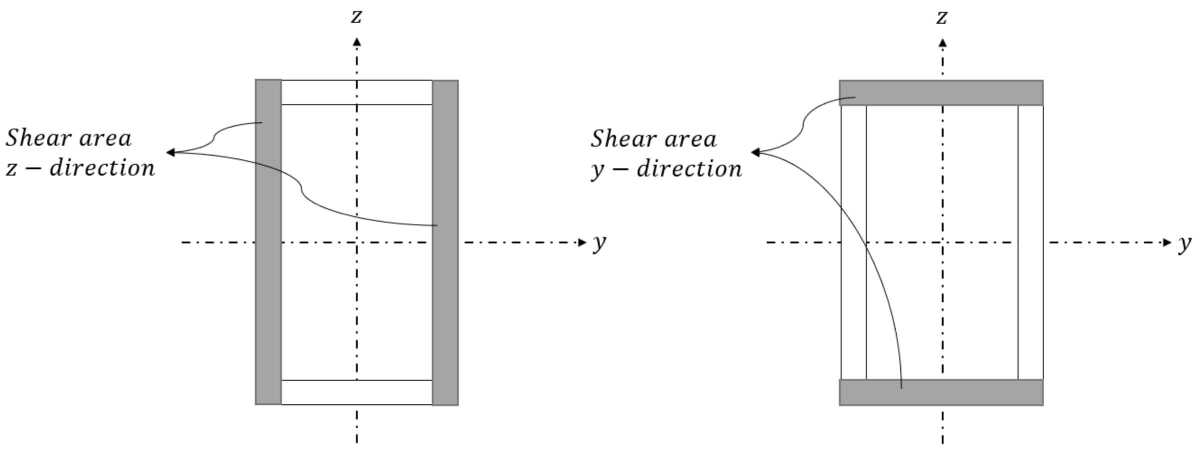
| | | |
|---|---|---|
| **Torsion: distance to neutral axis** | $z_{over} = D/2$<br>$y_{over} = D/2$<br>$z_{under} = D/2$<br>$y_{under} = D/2$ | |
| **Shear area** | $A_{vy} = \frac{2}{3}A$<br>$A_{vz} = \frac{2}{3}A$ | |
| **Shear constant, Kappa** | $\kappa_y = \frac{A}{A_{vy}}$<br>$\kappa_z = \frac{A}{A_{vz}}$ | |

## 2.2 Hollow rectangular



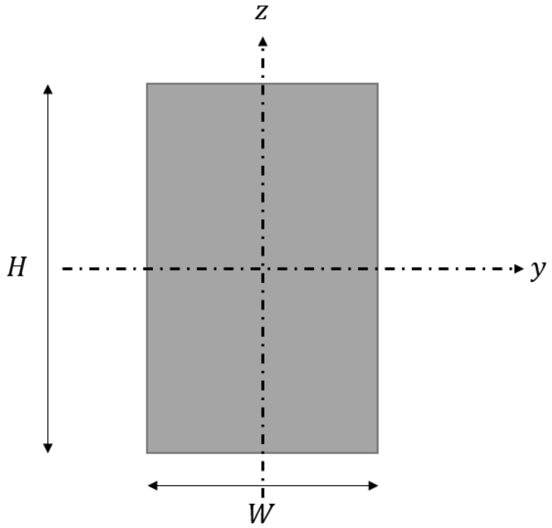



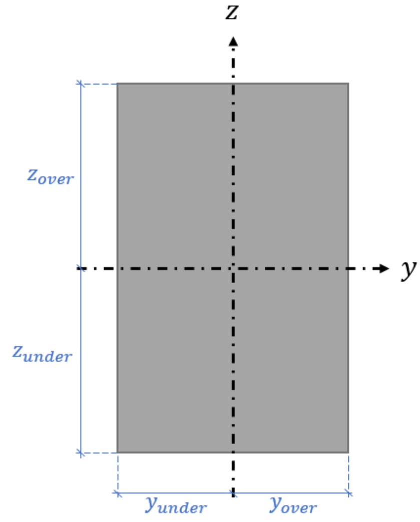

| Description | Formula | Comment |
|---|---|---|
| **Cross-sectional area** | $A = H \cdot W - (H - 2t) \cdot (W - 2t)$ | |
| **Second moment of area due to bending** | $I_y = \frac{WH^3}{12} - \frac{(W-2t)(H-2t)^3}{12}$ <br> $I_z = \frac{W^3H}{12} - \frac{(W-2t)^3(H-2t)}{12}$ | |
| **Second moment of area due to torsion** | $I_t = \frac{2(W-t)^2(H-t)^2t}{H+W-2t}$ | For references see e.g. (Johannesen, 2002). |
| **Bending: distance to neutral axis** | $z_{over} = H/2$ <br> $y_{over} = W/2$ <br> $z_{under} = H/2$ <br> $y_{under} = W/2$ | |
| **Torsion: distance to neutral axis** | $z_{over} = H/2$ <br> $y_{over} = W/2$ <br> $z_{under} = H/2$ <br> $y_{under} = W/2$ | |
| **Shear area** | $A_{vy} = 2 \cdot W \cdot t$ <br> $A_{vz} = 2 \cdot H \cdot t$ | See figure below for illustration of shear area. |
| **Shear constant, Kappa** | $\kappa_y = \frac{A}{A_{vy}}$ <br> $\kappa_z = \frac{A}{A_{vz}}$ | |



z
Shear area
z − direction
y
z
Shear area
y − direction
y

## 2.3 Massive rectangular

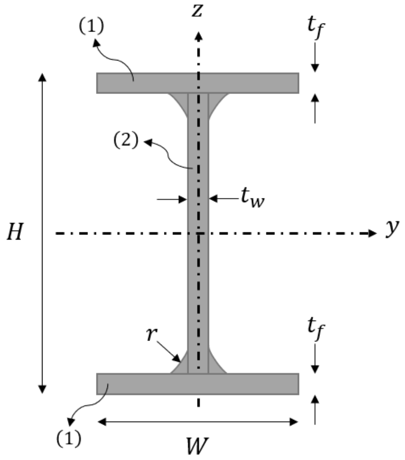


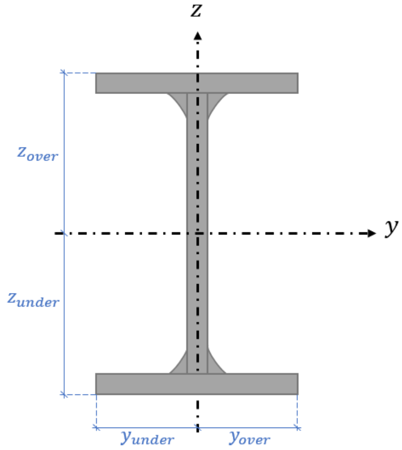




| Description | Formula | Comment |
|---|---|---|
| **Cross-sectional area** | $A = H \cdot W$ | |
| **Second moment of area due to bending** | $I_y = \frac{WH^3}{12}$ <br> $I_z = \frac{W^3H}{12}$ | |
| **Second moment of area due to torsion** | $I_t = HW^3\left(\frac{1}{3} - 0.21\frac{W}{H}\left(1 - \frac{W^4}{12H^4}\right)\right)$ | Valid for $H > W$. For reference see e.g. (Larsen, 1990) Ch. 5.2.3. |
| **Bending: distance to neutral axis** | $z_{over} = H/2$ <br> $y_{over} = W/2$ <br> $z_{under} = H/2$ <br> $y_{under} = W/2$ | |
| **Torsion: distance to neutral axis** | $z_{over} = H/2$ <br> $y_{over} = W/2$ <br> $z_{under} = H/2$ <br> $y_{under} = W/2$ | |
| **Shear area** | $A_{vy} = \frac{5}{6}W \cdot H$ <br> $A_{vz} = \frac{5}{6}W \cdot H$ | |

| | | |
|---|---|---|
| **Shear constant, Kappa** | $\kappa_y = \frac{A}{A_{vy}}$<br>$\kappa_z = \frac{A}{A_{vz}}$ | |

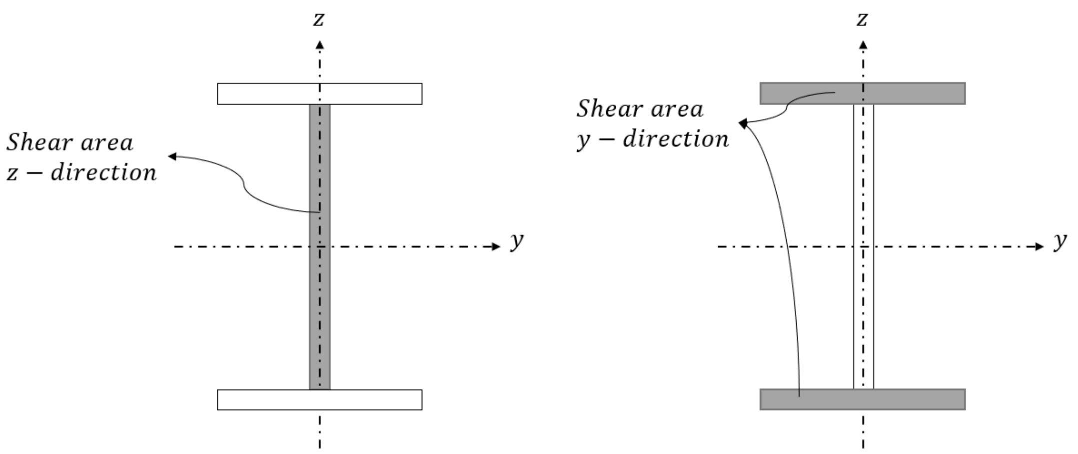
## 2.4 I-beam (length of Bottom flange = Top flange)



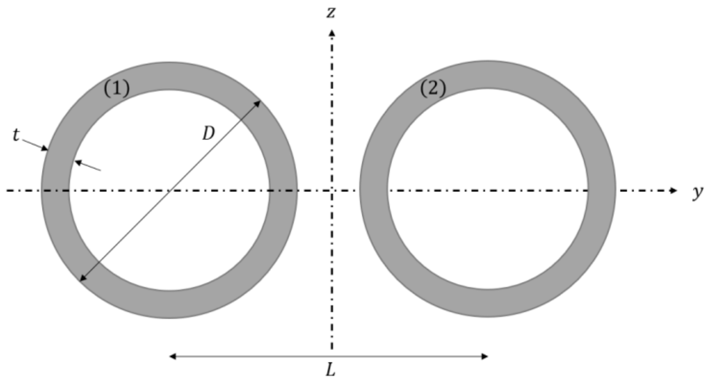



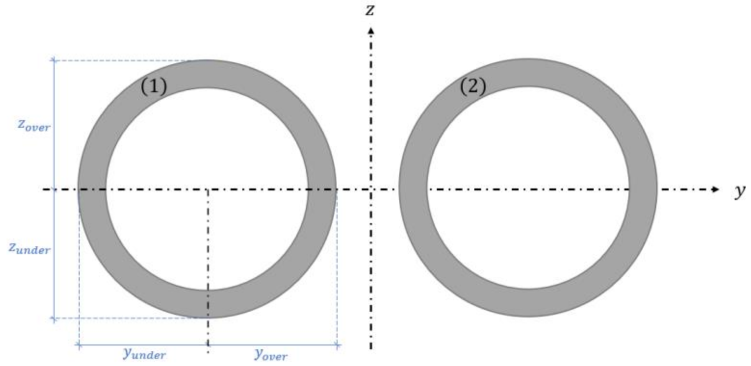

| Description | Formula | Comment |
|---|---|---|
| **Cross-sectional area** | $A = 2 \cdot A_{(1)} + A_{(2)}$<br>$= 2 \cdot W t_f + \left(H - 2 \cdot t_f\right) \cdot t_w$ | |
| **Second moment of area due to bending** | $I_{y(1)} = \frac{W t_f^3}{12}$<br>$I_{y(2)} = \frac{t_w\left(H - 2 \cdot t_f\right)^3}{12}$<br>$z_1 = \frac{1}{2}\left(H - t_f\right), \quad z_2 = 0$<br>$I_y = 2 \cdot \left(I_{y(1)} + A_{(1)} z_1^2\right) + I_{y(2)} + A_{(2)} z_2^2$<br><br>$I_{z(1)} = \frac{W^3 t_f}{12}$<br>$I_{z(2)} = \frac{t_w^3\left(H - 2 \cdot t_f\right)}{12}$<br>$I_z = 2 \cdot I_{z(1)} + I_{z(2)}$ | The second moment of area due to bending is calculated applying the parallel axis theorem (or Steiner's theorem). |

| | | |
|---|---|---|
| **Second moment of area due to torsion** | $h = H - 2 \cdot t_f$<br>$d = \frac{(t_f + r)^2 + t_w(r + 0.25t_w)}{2 \cdot r + t_f}$<br>$a = t_w + \left(\frac{4-\pi}{2}\right) \cdot \left(\frac{r^2}{d - t_f}\right)$<br>$\alpha = 0.46 - \frac{1}{2} \cdot \left(\frac{d}{a} - 1.15\right)^2$<br>$I_t = \frac{2}{3} t_f^3 (W - a) + \frac{1}{3} t_w^3 (h - 2d) + \alpha \cdot a \cdot d^3$ | The calculation of second moment of area due to torsion is based on empirical data, see e.g. (Fernando, 2022). |
| **Bending: distance to neutral axis** | $z_{over} = H/2$<br>$y_{over} = W/2$<br>$z_{under} = H/2$<br>$y_{under} = W/2$ | |
| **Torsion: distance to neutral axis** | $z_{over} = t_f$<br>$y_{over} = t_w$<br>$z_{under} = t_f$<br>$y_{under} = t_w$ | |
| **Shear area** | $A_{vy} = 2 \cdot W \cdot t_f$<br>$A_{vz} = (H - 2 \cdot t_f) \cdot t_w$ | See figure below for illustration of shear area. |
| **Shear constant, Kappa** | $\kappa_y = \frac{A}{A_{vy}}$<br>$\kappa_z = \frac{A}{A_{vz}}$ | |



## 2.5 Double tube




| Description | Formula | Comment |
|---|---|---|
| **Inner diameter of tube** | $d = D - 2 \cdot t$ | |
| **Cross-sectional area** | $A_{(1)} = A_{(2)} = \pi \left(\frac{D}{2}\right)^2 - \pi \left(\frac{d}{2}\right)^2$<br>$A = A_{(1)} + A_{(2)}$ | |
| **Second moment of area due to bending** | $I_{y(1)} = I_{y(2)} = \frac{\pi}{64}(D^4 - d^4)$<br>$I_y = I_{y(1)} + I_{y(2)}$<br>$I_{z(1)} = I_{z(2)} = \frac{\pi}{64}(D^4 - d^4)$<br>$I_z = I_{z(1)} + I_{z(2)}$ | For reference, see e.g. (Johannesen, 2002). |
| **Second moment of area due to torsion** | $I_t = I_y + I_z$ | |
| **Bending: distance to neutral axis** | $z_{over} = D/2$<br>$y_{over} = D/2$<br>$z_{under} = D/2$<br>$y_{under} = D/2$ | |

| | | |
|---|---|---|
| **Torsion: distance to neutral axis** | $z_{over} = D/2$<br>$y_{over} = D/2$<br>$z_{under} = D/2$<br>$y_{under} = D/2$ | |
| **Shear area** | $A_{vy} = \frac{2}{3}A$<br>$A_{vz} = \frac{2}{3}A$ | |
| **Shear constant, Kappa** | $\kappa_y = \frac{A}{A_{vy}}$<br>$\kappa_z = \frac{A}{A_{vz}}$ | |

# 3 Validation

This section presents results from validation of the cross sections. The validation is performed for AquaSim version 2.17.2.

## 3.1 Tube

Input parameters is presented in Table 1. Results and comparison are presented in Table 2.

*Table 1 Input parameters*

| Description | Abbreviation | Value |
|---|---|---|
| **Outer diameter** | $D$ | 300 mm |
| **Thickness** | $t$ | 20 mm |

*Table 2 Results and comparison*

| Parameter | Abbreviation | Analytical | AquaSim (Wizard) | Comparison [%] |
|---|---|---|---|---|
| **Cross sectional area** | $A$ | 1.7593E-02 m2 | 1.7593E-02 m2 | 0.00 |
| **Second moment of area, bending** | $I_y$ | 1.7229E-04 m4 | 1.7229E-04 m4 | 0.00 |
| | $I_z$ | 1.7229E-04 m4 | 1.7229E-04 m4 | 0.00 |
| **Second moment of area, torsion** | $I_t$ | 3.4658E-04 m4 | 3.4658E-04 m4 | 0.00 |
| **Distance from neutral axis, bending** | $z_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $z_{under}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{under}$ | 0.15 m | 0.15 m | 0.00 |
| **Distance from neutral axis, torsion** | $z_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $z_{under}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{under}$ | 0.15 m | 0.15 m | 0.00 |
| **Shear constant, Kappa** | $\kappa_y$ | 1.50 | 1.50 | 0.00 |
| | $\kappa_z$ | 1.50 | 1.50 | 0.00 |

## 3.2 Hollow rectangular

Input parameters is presented in Table 3. Results and comparison are presented in Table 4.

*Table 3 Input parameters*

| Description | Abbreviation | Value |
|---|---|---|
| **Height** | $H$ | 500 mm |
| **Width** | $W$ | 200 mm |
| **Thickness** | $t$ | 20 mm |

*Table 4 Results and comparison*

| Parameter | Abbreviation | Analytical | AquaSim (Wizard) | Comparison [%] |
|---|---|---|---|---|
| **Cross sectional area** | $A$ | 2.6400E-2 m2 | 2.6400E-2 m2 | 0.00 |
| **Second moment of area, bending** | $I_y$ | 7.8552E-4 m4 | 7.8552E-4 m4 | 0.00 |
| | $I_z$ | 1.7632E-4 m4 | 1.7632E-4 m4 | 0.00 |
| **Second moment of area, torsion** | $I_t$ | 4.5242E-4 m4 | 4.5242E-4 m4 | 0.00 |
| **Distance from neutral axis, bending** | $z_{over}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{over}$ | 0.10 m | 0.10 m | 0.00 |
| | $z_{under}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{under}$ | 0.10 m | 0.10 m | 0.00 |
| **Distance from neutral axis, torsion** | $z_{over}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{over}$ | 0.10 m | 0.10 m | 0.00 |
| | $z_{under}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{under}$ | 0.10 m | 0.10 m | 0.00 |
| **Shear constant, Kappa** | $\kappa_y$ | 3.30 | 3.30 | 0.00 |
| | $\kappa_z$ | 1.32 | 1.32 | 0.00 |

## 3.3 Massive rectangular

Input parameters is presented in Table 5. Results and comparison are presented in Table 6.

*Table 5 Input parameters*

| Description | Abbreviation | Value |
|---|---|---|
| **Height** | $H$ | 500 mm |
| **Width** | $W$ | 200 mm |

*Table 6 Results and comparison*

| Parameter | Abbreviation | Analytical | AquaSim (Wizard) | Comparison [%] |
|---|---|---|---|---|
| **Cross sectional area** | $A$ | 1.0000E-1 m2 | 1.0000E-1 m2 | 0.00 |
| **Second moment of area, bending** | $I_y$ | 2.0833E-3 m4 | 2.0833E-3 m4 | 0.00 |
| | $I_z$ | 3.3333E-4 m4 | 3.3333E-4 m4 | 0.00 |
| **Second moment of area, torsion** | $I_t$ | 9.9805E-4 m4 | 9.9805E-4 m4 | 0.00 |
| **Distance from neutral axis, bending** | $z_{over}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{over}$ | 0.10 m | 0.10 m | 0.00 |
| | $z_{under}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{under}$ | 0.10 m | 0.10 m | 0.00 |
| **Distance from neutral axis, torsion** | $z_{over}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{over}$ | 0.10 m | 0.10 m | 0.00 |
| | $z_{under}$ | 0.25 m | 0.25 m | 0.00 |
| | $y_{under}$ | 0.10 m | 0.10 m | 0.00 |
| **Shear constant, Kappa** | $\kappa_y$ | 1.20 | 1.20 | 0.00 |
| | $\kappa_z$ | 1.20 | 1.20 | 0.00 |

## 3.4 I-beam

Input parameters is presented in Table 7. Results and comparison are presented in Table 8.

*Table 7 Input parameters*

| Description | Abbreviation | Value |
|---|---|---|
| **Flange width** | $W$ | 250 mm |
| **Flange thickness** | $t_f$ | 38 mm |
| **Height** | $H$ | 300 mm |
| **Web thickness** | $t_w$ | 25 mm |
| **Radius** | $r$ | 20 mm |

*Table 8 Results and comparison*

| Parameter | Abbreviation | Analytical | AquaSim (Wizard) | Comparison [%] |
|---|---|---|---|---|
| **Cross sectional area** | $A$ | 2.4600E-2 m2 | 2.4600E-2 m2 | 0.00 |
| **Second moment of area, bending** | $I_y$ | 3.5176E-4 m4 | 3.5176E-4 m4 | 0.00 |
| | $I_z$ | 9.9250E-5 m4 | 9.9250E-5 m4 | 0.00 |
| **Second moment of area, torsion** | $I_t$ | 1.0647E-5 m4 | 1.0647E-5 m4 | 0.00 |
| **Distance from neutral axis, bending** | $z_{over}$ | 0.150 m | 0.150 m | 0.00 |
| | $y_{over}$ | 0.125 m | 0.125 m | 0.00 |
| | $z_{under}$ | 0.150 m | 0.150 m | 0.00 |
| | $y_{under}$ | 0.125 m | 0.125 m | 0.00 |
| **Distance from neutral axis, torsion** | $z_{over}$ | 0.038 m | 0.038 m | 0.00 |
| | $y_{over}$ | 0.025 m | 0.025 m | 0.00 |
| | $z_{under}$ | 0.038 m | 0.038 m | 0.00 |
| | $y_{under}$ | 0.025 m | 0.025 m | 0.00 |
| **Shear constant, Kappa** | $\kappa_y$ | 1.29 | 1.29 | 0.00 |
| | $\kappa_z$ | 4.39 | 4.39 | 0.00 |

## 3.5 Double tube

Input parameters is presented in Table 9. Results and comparison are presented in Table 10.

*Table 9 Input parameters*

| Description | Abbreviation | Value |
|---|---|---|
| **Outer diameter** | $D$ | 300 mm |
| **Thickness** | $t$ | 20 mm |

*Table 10 Results and comparison*

| Parameter | Abbreviation | Analytical | AquaSim (Wizard) | Comparison [%] |
|---|---|---|---|---|
| **Cross sectional area** | $A$ | 3.5186E-2 m2 | 3.5186E-2 m2 | 0.00 |
| **Second moment of area, bending** | $I_y$ | 3.4658E-4 m4 | 3.4658E-4 m4 | 0.00 |
| | $I_z$ | 3.4658E-4 m4 | 3.4658E-4 m4 | 0.00 |
| **Second moment of area, torsion** | $I_t$ | 6.9316E-4 m4 | 6.9316E-4 m4 | 0.00 |
| **Distance from neutral axis, bending** | $z_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $z_{under}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{under}$ | 0.15 m | 0.15 m | 0.00 |
| **Distance from neutral axis, torsion** | $z_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{over}$ | 0.15 m | 0.15 m | 0.00 |
| | $z_{under}$ | 0.15 m | 0.15 m | 0.00 |
| | $y_{under}$ | 0.15 m | 0.15 m | 0.00 |
| **Shear constant, Kappa** | $\kappa_y$ | 1.50 | 1.50 | 0.00 |
| | $\kappa_z$ | 1.50 | 1.50 | 0.00 |

# 4 Conclusions

The validation cases show that there is good correspondence between analytical calculations of cross sectional properties and AquaSim.

# 5 References

Fernando, L. (2022, 08 11). *Torsional Constant Calculator*. Retrieved from Omni Calculator: https://www.omnicalculator.com/physics/torsional-constant

Johannesen, J. (2002). *Tekniske tabeller.* Capellen.

Larsen, P. K. (1990). *Dimensjonering av stålkonstruksjoner.* Tapir.