Calculation of loads on turbine blades using BEM theory

TR-FOU-100004-5

Revision 4

Report no.:	TR-FOU-100004-5		
Date of this revision:	30.11.2020		
Number of pages:	28		
Distribution:	Open		
Author:	Are Johan Berstad	Keywords:	BEM, wind turbine, controller

Summary:

This document follows *Aerodynamics of Wind Turbines* by Martin O.L. Hansen (Hansen, 2008). Lift elements have been used in AquaSim for a long time. Such elements are based on calculating drag and lift to beam elements by the cross-flow principle. That is consistent with (Hansen, 2008) ch. 2. Then, corrections according to Chapter 4 to obtain the BEM theory is presented.

Revision no.	Date	Author	Verified by	Description
4	01.02.2024	AJB	ISH	Sec. 3, 4.1-4.3, 4.5-4.7 added
3	04.12.2023	AJB	ISH	Updates Figure 9 and sec. 5
2	16.10.2023	AJB	ISH	Updates Figure 4-6 and 9
1	11.10.2023	AJB	ISH	First revision

Content

1 Introduction

This document follows *Aerodynamics of wind turbines*, (Hansen, 2008). Lift elements have been used in AquaSim for å long time. Such elements are based on calculating drag and lift on beam elements by the cross-flow principle. That is consistent with (Hansen, 2008) Ch. 2. Then, corrections according to (Hansen, 2008) Ch. 4 to obtain BEM theory is conducted. (Hansen, 2008) Ch. 6 includes Prandtl's loss factor and the Glauert correction.

2 Theory

This section follows the (Hansen, 2008) layout for BEM method. Consider turbine blades as shown in Figure 1.

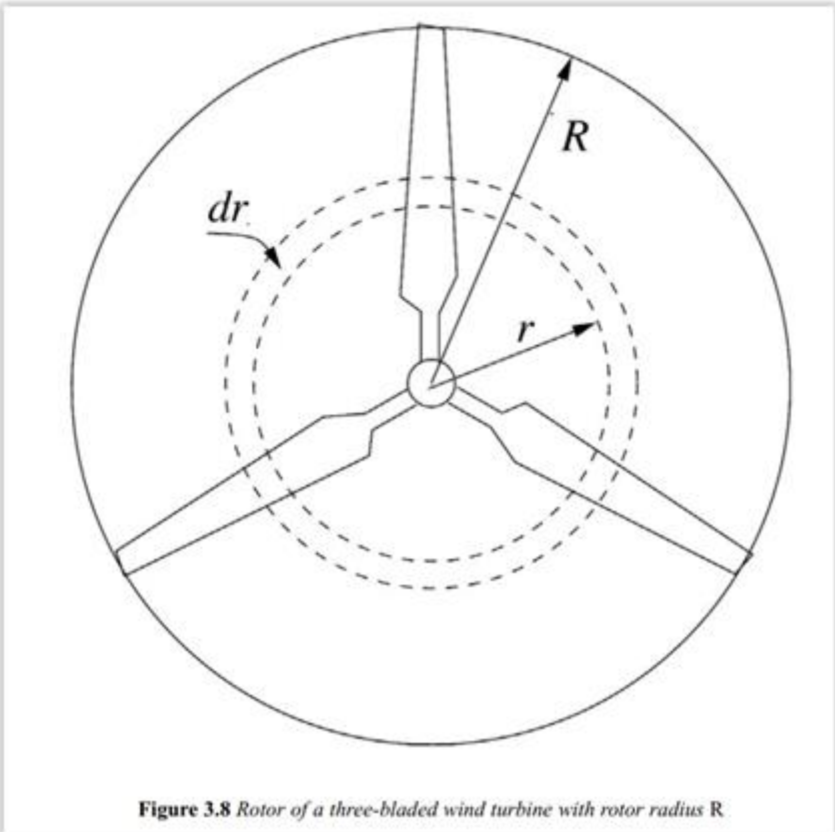

Figure 1 A strip dr of a turbine blade. (Hansen, 2008) Figure 3.8

The total radius of a turbine blade from the nave is R and the radius to a considered infinitesimal strip dr is r, as shown in Figure 1.

2.1 Drag and lift forces on 2D surfaces

The BEM means applying drag- and lift coefficients to the strip marked as dr in Figure 1 for a finite set of strips as seen in Figure 2. However, contrary to drag and lift applied to a non-rotating object a correction should be applied to the incident wind velocity and the rotational speed since the fact that the blades rotated means there is an effect loss.

Start out with considering a beam element in AquaSim with a local *x*- axis from node 1 to node 2. Figure 2 is then the local *yz*-plane of the beam element. Let the local *z*- axis be along the chord of the element to the right in Figure 2. Now, assume the cross flow principle to be valid such that we consider a 2D problem in the *yz*-plane. By definition, the drag force D to an object is the force parallel to the direction of the incident relative flow V_∞. This is normally described by:

$$F_D = D = C_d c \frac{\rho}{2} V_\infty |V_\infty|$$

where C_d is the drag coefficient in the direction of V_∞ (D in Figure 2), and in the lift force L is found as:

$$F_L = L = C_l c \frac{\rho}{2} V_\infty |V_\infty|$$

where C_l is the coefficient for the lift force (L in Figure 2). Lift force is perpendicular to the drag force.

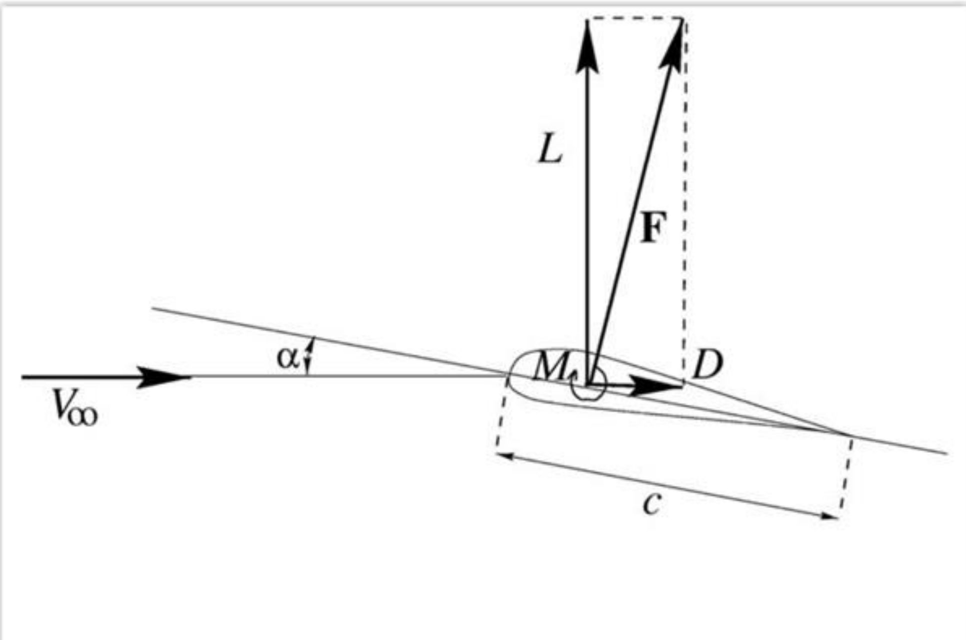

Figure 2.2 *Definition of lift and drag*

Figure 2 (Hansen, 2008) Figure 2.2

In Figure 2, α is the inflow angle relative to the definition on the table where drag, lift and moment coefficients are given as a function of α.

2.2 BEM theory. Modification of drag and lift due to blade rotation

For a turbine blade in steady state, the two most important components of the V_∞ is the wind velocity and the velocity caused by the blade rotating around the nave. Consider a case where there is a steady undisturbed wind with no wind gust and a velocity V_0 and steady rotation of a turbine blade ωr, the cross-flow inflow velocity seen by the considered cross section of the blade, V_∞ will then be:

$$V_\infty = V_0 - \omega r$$

Equation 1

where ω is the angular velocity of the blade and r is the distance from the nave to the cross section of the blade. V_0 is the inflow undisturbed wind velocity.

The blade rotating around the nave introduces a 3D effect causing a pure cross flow assessment to each strip of the blade to cause an inaccuracy and there will be some less effect and energy out-take by the turbine blades than what is assumed by a "raw" cross flow assessment. This is described by the BEM theory. The cross flow assessment width drag and lift coefficients described above is the base of the BEM theory, but some adjustments are carried out, these are:

1. Adjustment of inflow wind velocity stemming from accounting for conservation of momentum for the full circle, see (Hansen, 2008).
2. The Prandtl correction for tip loss.
3. The Glauert correction.

(Hansen, 2008) point 1 in the listing results in a modification in the inflow velocity be accounted for by introducing a correction to the inflow velocity as shown in Figure 3.

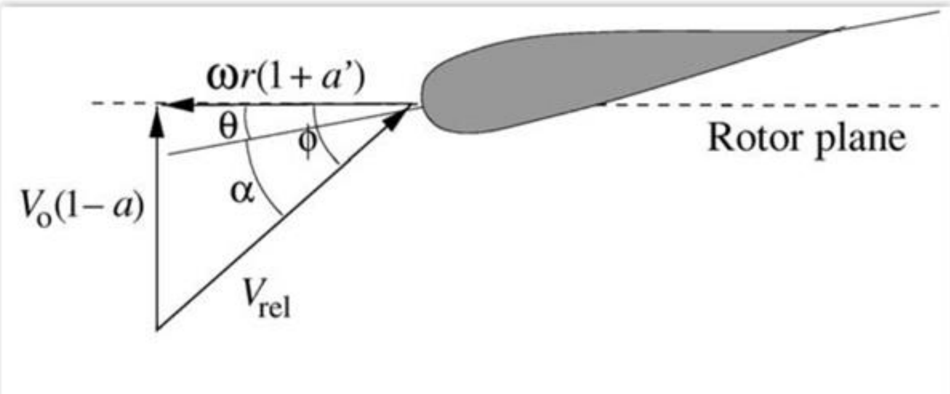

Figure 6.2 *Velocities at the rotor plane*

Figure 3 (Hansen, 2008) Figure 6.2

In Figure 3, the correction terms a and a' are introduced:

- a reduces the inflow velocity V_0.
- a' increase the velocity in the rotational direction.

The BEM theory is then using crossflow, but with relative velocity used as input to the flow corrected with a and a'. This means that the turbine blade will be less effective than predicted using a "raw" cross flow approach. a and a' are corrections that is to be applied to V_0 and ωr respectively to account for the 3D effect caused by the rotation.

Following (Hansen, 2008), a and a' in Figure 3 is found as:

$$a = \frac{1}{\frac{4F \cdot sin^2\phi}{\sigma C_n} + 1}$$

Equation 2

and

$$a' = \frac{1}{\frac{4F \cdot sin\phi \cdot cos\phi}{\sigma C_t} - 1}$$

Equation 3

This is according to (Hansen, 2008) formula 6.35 and 6.36. The coefficients C_n and C_t is found as:

$$C_n = C_l \cos\phi + C_d \sin\phi$$

Equation 4

$$C_t = C_l \sin\phi - C_d \cos\phi$$

Equation 5

where C_l and C_d is the lift and drag coefficient of the cross section. ϕ is the angle as seen in Figure 3. The appropriate C_l and C_d to be used in is found at angle α in Figure 3. θ then denotes how much the 0-inflow angle of this part of the blade is tilted relative to the direction of rotation about the nave.

σ is the solidity of the turbine blades. This is the part of the circle which at a given time interval is covered with blades in the direction of the axis:

$$\sigma(r) = \frac{c(r)B}{2\pi r}$$

Equation 6

where

- B is the number of blades,
- r is the distance from the considered cross section of the nave,
- $c(r)$ is the chord length of blade at r.

Let F be a correction factor for the "tip" effect, (Hansen, 2008) Eq. 6.33:

$$F = \frac{2}{\pi} cos^{-1}(e^{-f})$$

Equation 7

where

$$f = \frac{B}{2} \frac{(R - r)}{r \sin \phi}$$

where $B$and r are as defined before, and:

- R is the total radius of the rotor,
- ϕ is the angle according to Figure 3.

Equation 2 is valid for $a < a_{cr}$ where a typical value for a_{cr} is 1/3. For values $> a_{cr}$, the expression in Equation 8 may be used for a, whereas Equation 3 is still applied for a'.

$$a = \frac{1}{2}\left[2 + K(1 - 2a_{cr}) - \sqrt{(K(1 - 2a_{cr}) + 2)^2 + 4(Ka_{cr}^2 - 1)}\right]$$

Equation 8

Equation 8 is according to (Hansen, 2008) Eq. 6.44. Where:

$$K = \frac{4F \sin^2 \phi}{\sigma C_n}$$

Equation 9

2.3 Iteration scheme

With reference to Figure 3, this means that we will use Equation 2 or Equation 6 to adjust the inflow velocity V_0, and Equation 3 to adjust the rotational velocity. As seen from the equations we must apply an iteration scheme to find a and a', see (Hansen, 2008) pp. 50. In AquaSim, this iteration schemes goes as follows:

- Step 1: initialize a and a', typically $a = a' = 0$.
- Step 2: find the flow-angle ϕ and local angle of attack (Figure 3).
- Step 3: read $C_l(\alpha)$ and $C_d(\alpha)$ from the cross-flow drag- and lift table.
- Step 4: C_n and C_t is calculated from Equation 4 and Equation 5.
- Step 5: a is found from Equation 2 or Equation 6, and a' from Equation 3.
- Step 6: set a and a' to bound. If they are out bound go to step 7.
- Step 7: apply the a and a' as start-values and return to step 2. Then when the difference between a and a' in two consecutive steps are below the tolerance, or the number of iterations has reached max, a and a' are used for step 8.
- Step 8: in AquaSim, the relative velocity for a cross-section is used with the cross-flow principle to derive the drag and lift load to an element. The input relative velocity to the considered blade cross-section is adjusted by a in the nave direction, and with a' in the rotation-direction. Then drag and lift loads are calculated.

3 Control systems

A wide range of parameters are adjusted and controlled on a wind turbine. The basic principle for the turbine is that a torque resistance generates the electricity with wattage proportional to the torque resistance and rotational velocity. The most common parameters that is adjusted by the control system is:

- **Yaw:** this is the turbine angle relative to the wind direction. The control system adjusts the yaw angle to keep the turbine facing the wind. The control system typically applies a wind sensor to determine the wind direction.
- **Blade pitch:** is the angle of the blades relative to the wind. The control system adjusts the blade pitch to keep the turbine operating at its optimum efficiency. Torque sensors are commonly applied to determine the optimal blade pitch. For low wind velocities, constant pitch is common. When the wind is higher, the pitch is increased in order to prevent too high rotational velocities.
- **Torque resistance:** the generate effect is proportional to rotational velocity and torque resistance. When the wind is so low that the max generated effect is below max effect, the torque resistance is regulated to maximize the effect. Other concerns may also give input to torque resistance regulation, such as eigen periods of the system.

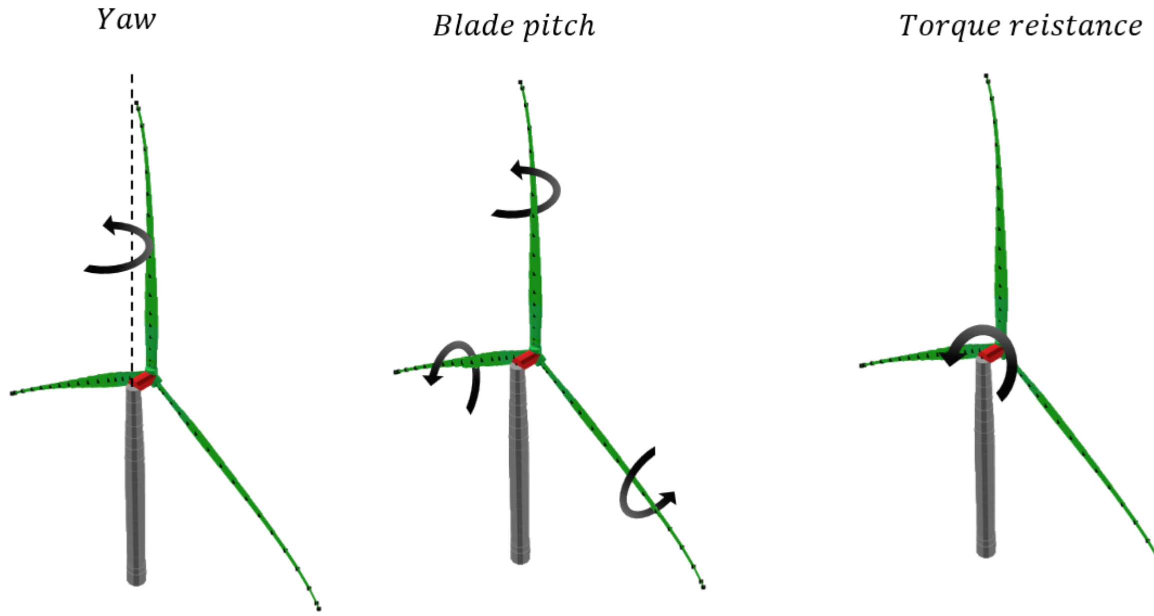

Figure 4

3.1 PI control system

The most common controllers are what is called *PI-controllers*. In a PI controller, "PI" stands for Proportional-Integral. A PI controller is a type of feedback control system commonly used in engineering and automation to regulate a process or system. It combines two fundamental control actions: proportional control (P) and integral control (I).

Proportional control (P): the proportional control component is based on the current error, which is the difference between the desired setpoint and the actual output or process variable. The controller responds to this error by applying a control action proportional to the error. In simple terms, the larger the error, the stronger the corrective action. Mathematically, the proportional control output (P) is given by:

$$P = K_P \cdot error$$

where K_P is the proportional gain, a tuning parameter that determines the strength of the proportional response.

Integral Control (I): The integral control component addresses any accumulated error over time. It considers the integral of the error and applies a control action to eliminate the steady-state error. This helps in addressing issues related to long-term deviations from the setpoint. Mathematically, the integral control output (I) is given by:

$$I = K_I \cdot \int (error)\, dt$$

where K_I is the integral gain, a tuning parameter that determines how aggressively the controller eliminates accumulated error. The output of a PI-controller is the sum of the proportional and integral components:

$$Controller\ output = P + I$$

The combination of proportional and integral actions allows a PI-controller to provide a fast initial response to changes in the system (proportional action) while eliminating any steady-state error that may persist over time (integral action). PI-controllers are widely used due to their simplicity, effectiveness, and ease of tuning for a variety of control applications, including those in wind turbine control systems.

Note that PI-controllers in general are mostly established for wind turbines attached to substructures fixed to ground. When applying a PI (Proportional-Integral) controller to a floating system, such as a floating platform for offshore wind turbines, there are specific challenges and considerations that need to be addressed. The dynamic nature of floating systems introduces complexities that may require additional control strategies or modifications to the PI controller. Aspects includes but may not be limited to:

- System Dynamics and Nonlinearities: Floating systems are inherently dynamic and nonlinear due to the complex interactions between the platform, moorings, and the surrounding water. The PI controller, which is a linear controller, may face challenges in effectively handling the nonlinearities and time-varying characteristics of the floating platform.

- Wave and Wind Interactions: Floating platforms are subject to wave and wind interactions, which can introduce disturbances affecting the system's motion. The PI controller may need to be robust enough to handle these disturbances and adjust control actions accordingly.
- Variable Operating Conditions: Floating platforms may experience varying operating conditions, such as changes in wave height, wind speed, and sea state. The PI controller may require adaptive tuning or the incorporation of additional control strategies to ensure optimal performance across a range of conditions.
- System Coupling: The coupling between different degrees of freedom in a floating system can complicate the control task. For instance, the motion of the platform in one direction may affect its behaviour in other directions. Advanced control techniques, beyond standard PI control, may be necessary to address these coupled dynamics.
- Resonance and Oscillations: Floating structures are susceptible to resonant frequencies and oscillations induced by external forces. The PI controller's tuning parameters may need careful consideration to avoid amplifying resonant behaviour.
- Sensor Integration and Measurement Noise: Accurate sensor measurements are crucial for the proper functioning of a PI controller. Floating systems may experience additional challenges related to sensor integration, increased measurement noise due to motion, and potential signal delays.
- Modeling Accuracy: The accuracy of the mathematical model used in the control system is crucial. Floating systems can exhibit complex and nonlinear behaviours that might not be fully captured by simplified models. Controller performance may be improved with more accurate and detailed models.
- Design considerations: How well can controllers be trusted in extreme design cases. For design, the worst combinations of wind, currents, and waves as well as operating conditions should be assessed. In such conditions the risk of errors such as pitch regulator being locked for some reason is at its highest. These are often referred to as accidental conditions.

To address these challenges, advanced control strategies, such as model predictive control, adaptive control, or nonlinear control, may be considered. Additionally, thorough testing, simulation studies, and validation against real-world data are essential to ensure the robustness and effectiveness of the control system on a floating platform.

In order to obtain good controller for floating systems control engineering and offshore engineering is often necessary to design and implement a control system that meets the specific requirements of a floating wind turbine or other floating structure. This is the background for the possibilities in AquaSim ranging from fixed control parameters to user defined and programmed logic through a Python interface.

3.2 Operation and rated wind speed

The two basic operating conditions in wind turbine operation are often described as follows:

- **Below Rated Power Operation (Subrated or Partial Load):**
  this is the condition when the wind speed is relatively low, and the wind turbine is operating below its rated power. In this range, the goal is to extract as much power as possible from the wind while avoiding overloading the turbine. The pitch angle of the turbine blades may be adjusted to optimize energy capture.
- **Above Rated Power Operation (Superrated or Full Load):**
  this condition occurs when the wind speed is high, and the wind turbine is operating at or near its rated power. At this point, the turbine is extracting the maximum amount of power it is designed for. To prevent overloading the turbine and its components, control strategies are employed. One common approach is to pitch the blades out of the wind to reduce the aerodynamic forces and limit the power capture.

These two conditions represent the two ends of the operational spectrum for a wind turbine. The transition between these conditions is managed by the turbine's control system, which includes pitch control mechanisms, generator torque control, and other strategies to ensure safe and efficient operation across varying wind speeds. The specific wind speed at which the turbine transitions from subrated to superrated operation is typically referred to as the "rated wind speed."

When a wind turbine is operating very close to its rated wind speed, the transition from below-rated to above-rated operation is managed by the turbine's control system. This transition involves adjusting various parameters to optimize the turbine's performance, capture as much energy as possible, and prevent overloading the turbine components. The specific strategies employed during this transition can vary between different turbine models and is more complicated for floating systems than for fixed systems.

4 Implementation to AquaSim

In AquaSim, the different control systems are referred to as *Controller*. The next chapters describe the available controllers in AquaSim:

- AquaSim Quick Design
- Internal
- ROSCO (Python integration)

4.1 AquaSim Quick design

A simplified controller with the aim of quickly guiding the turbine blades into the steady state condition. The following parameters are **constant** for the *AquaSim Quick Design*:

- Yaw angle
- Pitch angle
- Torque resistance

If the user has specified a *Quick start velocity*, the turbine blades will start directly into this velocity when the dynamic part of the analysis starts. This controller is ideal to apply in cases where there is a need to determine the combined effects for wind, current and waves for a given state.

4.2 Internal

The *Internal* controller is a type of PI-controller. Having selected this controller type, the user must specify appropriate PI parameters. These are found in the Advanced section of the Generate turbine-tool, see Figure 5.

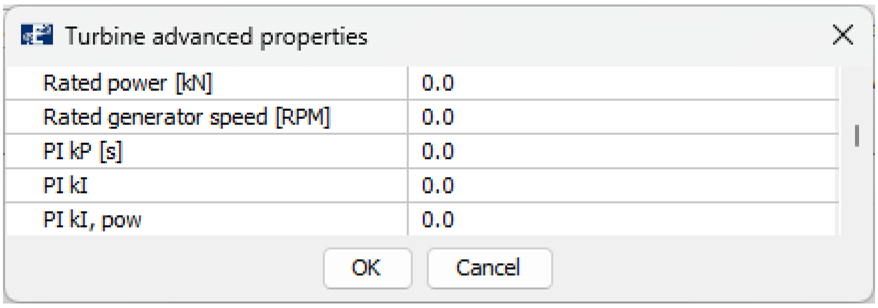

Figure 5 PI parameters to be defined in AquaSim

The PI controller is the one described by (Silva de Souza, et al., 2021). Note that this type of PI-controller is applicable for rated speed as presented in chapter 3.2.

When the wind is below rated wind speed, the relationship between torque and rotational velocity (angular speed) is influenced by the aerodynamic characteristics of the turbine blades. The torque generated by the wind turbine is a function of the wind speed, the pitch angle of the blades, and other factors. The torque-speed relationship normally depends on the aerodynamic design and control strategy of the turbine. Below the rated wind speed, the relationship between torque and rotational velocity may be linearized, assuming that the aerodynamic forces are approximately proportional to the square of the blade tip speed. The linearized torque-speed relationship can be expressed as follows:

$$T = a\omega + b\omega^2$$

where

- T is torque,
- ω is the rotational velocity,
- a and b are coefficients.

In this linearization, the term $a\omega$ represents the aerodynamic torque that is linear with respect to rotational velocity, and the term $b\omega^2$ represents the additional torque due to the quadratic relationship. As the wind speed increases and the turbine approaches its rated speed, the aerodynamic characteristics may change, and control strategies, such as pitch control, may be used to adjust the torque-speed relationship.

The specific form of the torque-speed relationship can vary between different wind turbine designs and manufacturers. Detailed turbine models and control system specifications provided by manufacturers typically include the equations or curves describing the torque-speed relationship for various wind conditions.

In AquaSim on can give in a resistance k_{br} based on any consideration and use the fixed quick-design in the further analysis.

4.3 ROSCO (Python integration)

The *ROSCO (Python integration)* controller is a type of PI-controller. Having selected this type, AquaSim offers interaction to Python as well as the NREL ROSCO-regulator. The users can themselves program the Python layer to use any chosen regulator. In this case a range of parameters are sent to Python and the user can program in Python as desired and send back data based on used controller. The NREL controller is included in as a possibility. Note that if this is to be utilised, some installation files are necessary.

4.4 Validation

Section 12 from (Silva de Souza, et al., 2021) Figure 66, has been chosen for validation.

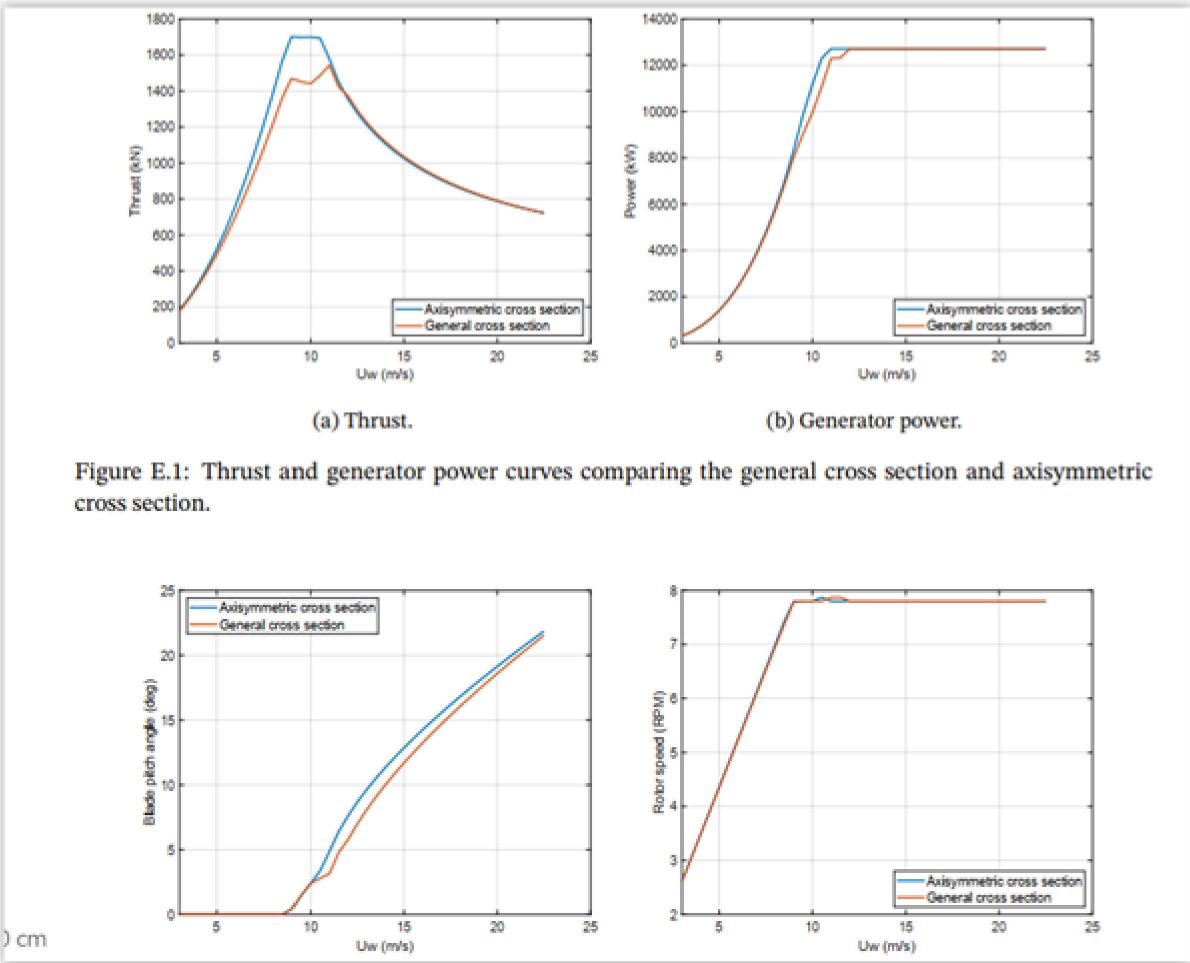

Figure 6 Results from (Silva de Souza, et al., 2021)

Analysis was carried out with nominal constant wind, and results are presented in Figure 7-Figure 12. As seen from the figures, results compare well.

4.4.1 Spin velocity

Figure 7 shows the nave spin velocity [RPM].

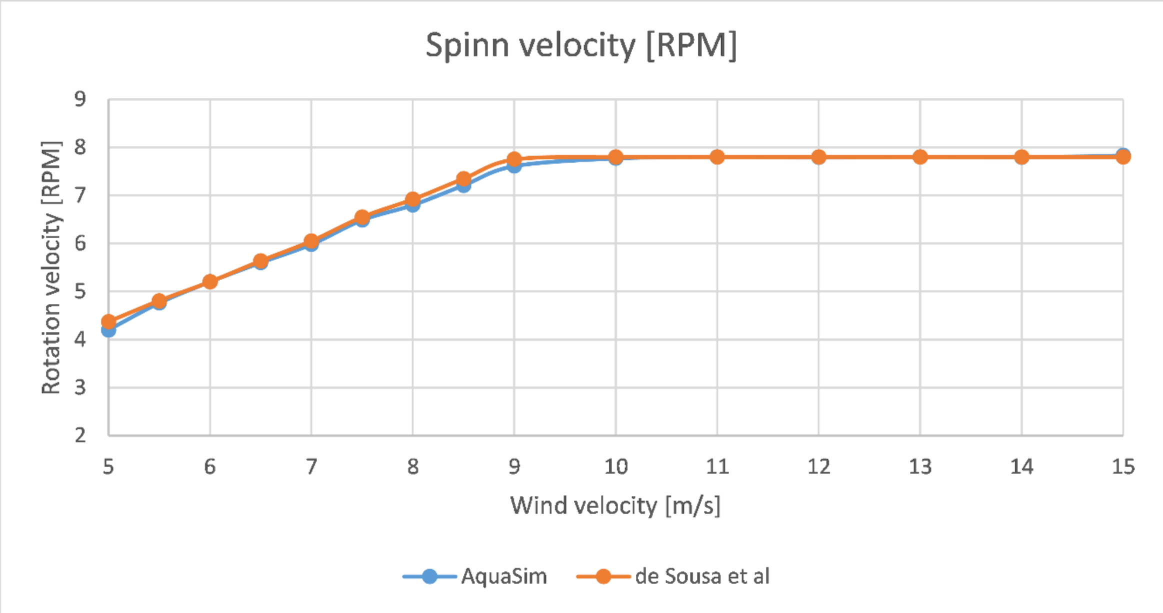

Figure 7 Naven spin velocity

4.4.2 Torque

Figure 8 shows comparison of torque moment. As seen from the figure the AquaSim analysis in this report is a little below in the critical area whereas it compares well in the other zones.

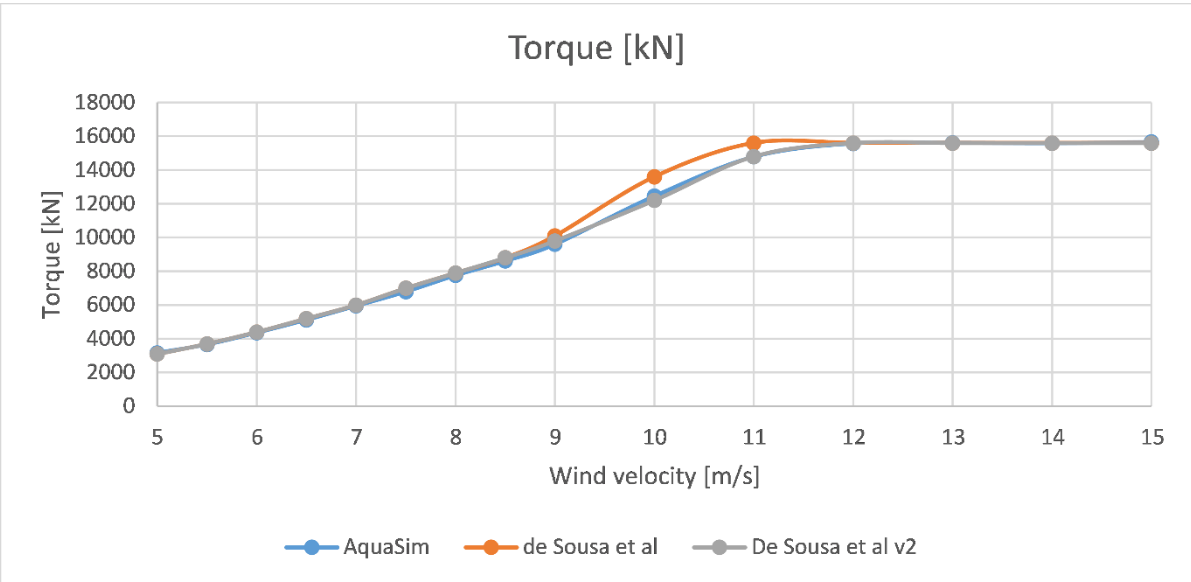

Figure 8 Torque

4.4.3 Generator power

Results for power is a function of torque and spin-velocity. Meaning the results are similar as for torque, in chapter 4.4.2.

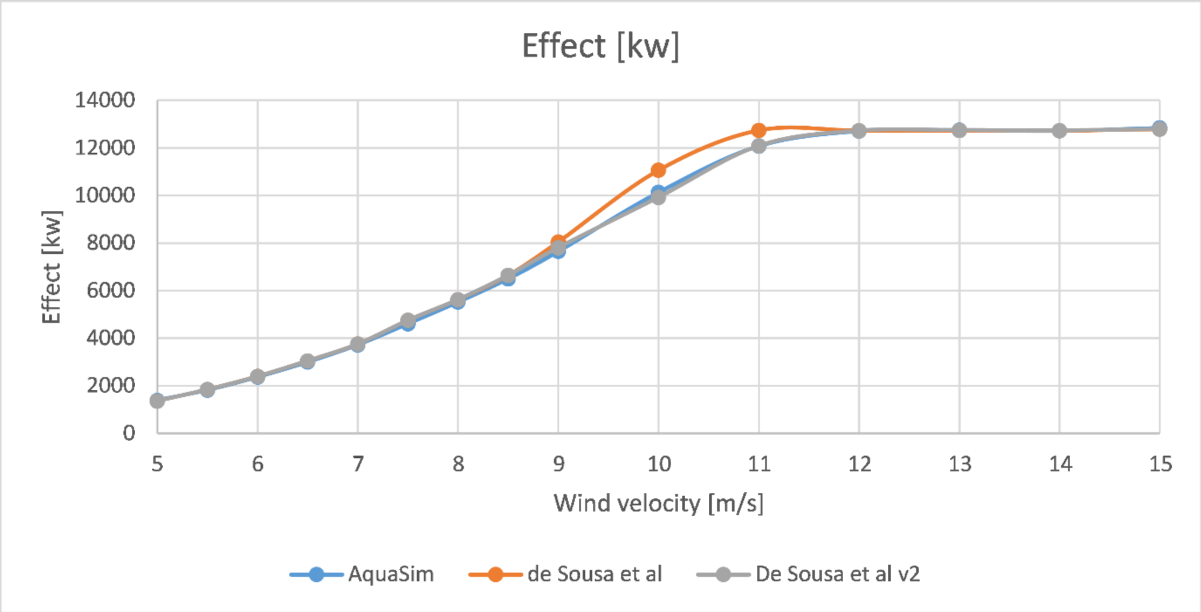

Figure 9 Power

4.4.4 Thrust

Figure 10 shows results comparison for thrust.

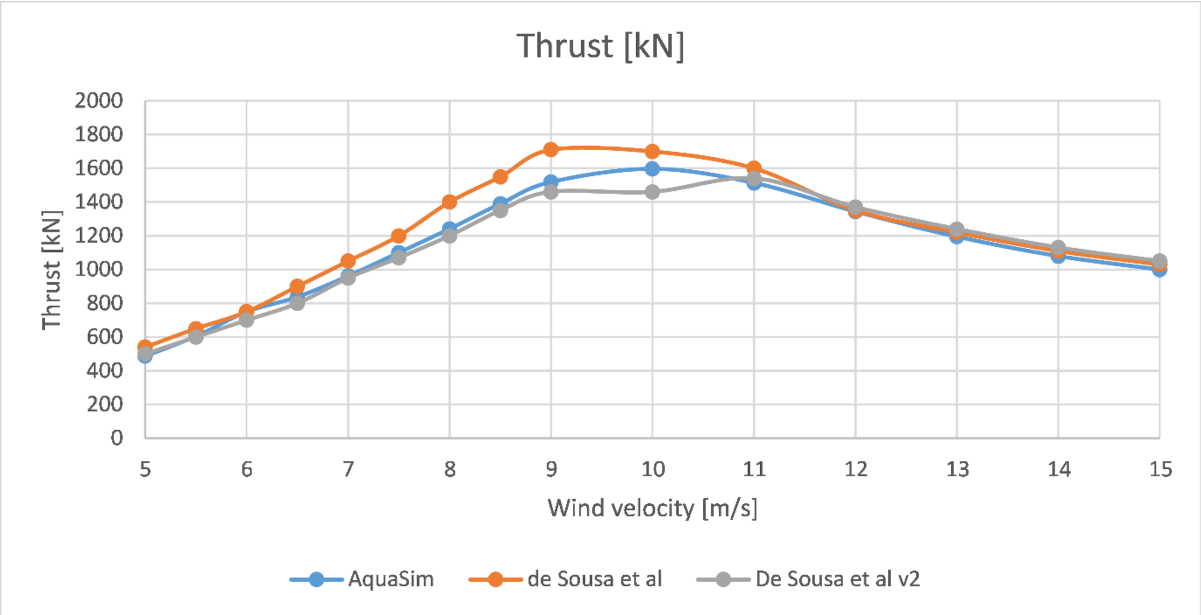

Figure 10 Thrust

4.4.5 Blade tip displacement

Figure 11 shows results for the blade tips displacement.

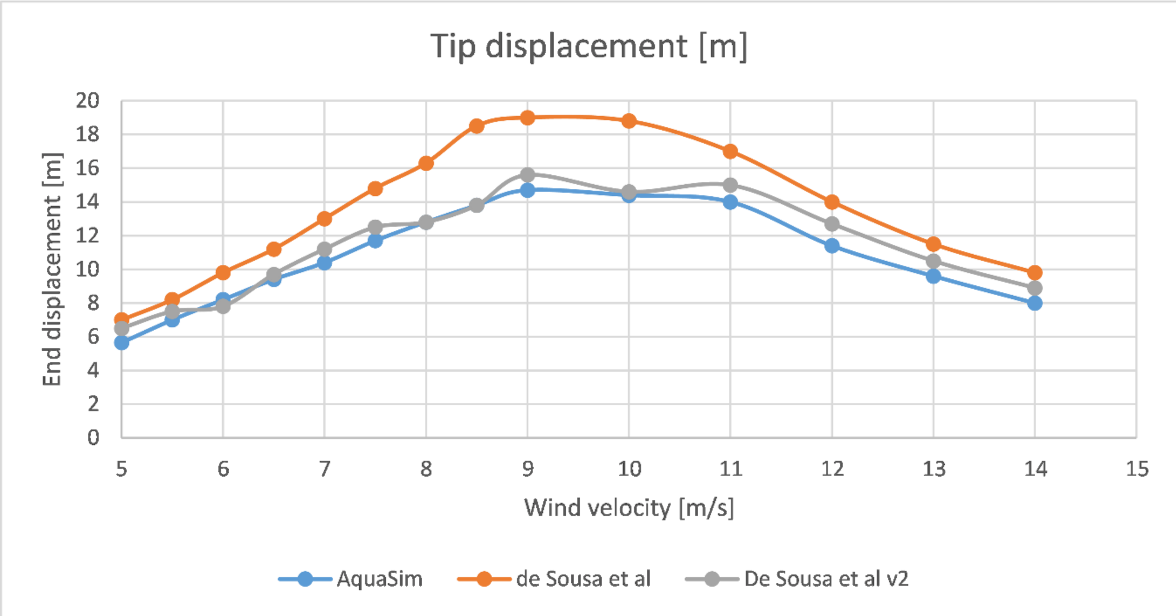

Figure 11 Blade tip displacement

4.4.6 Pitch

Figure 12 shows pitch angle of the blades.

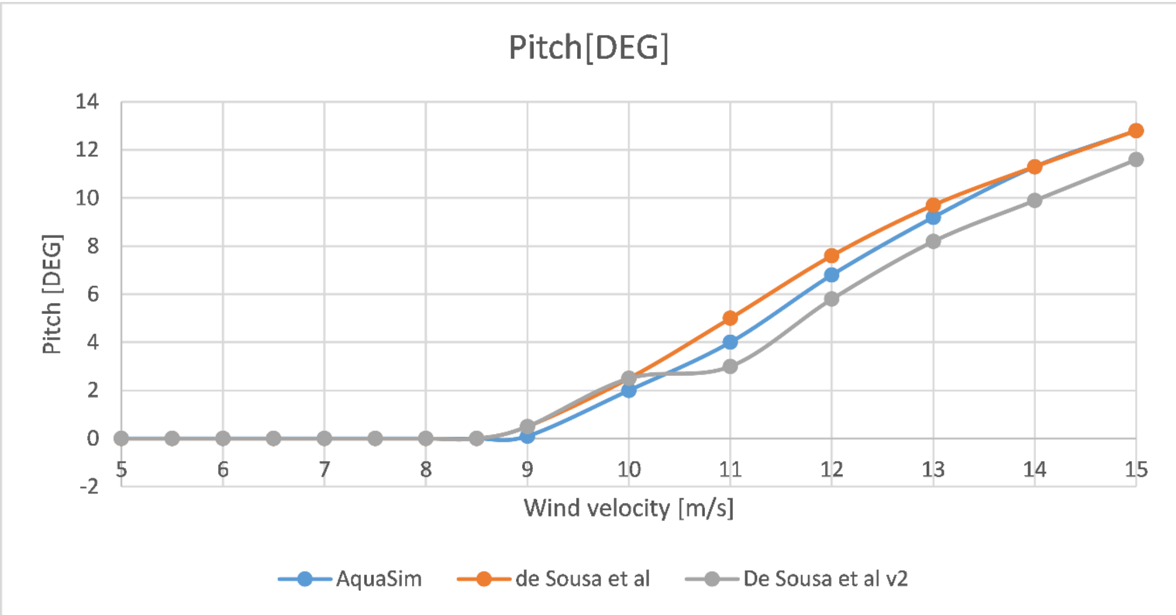

Figure 12 Pitch angle of blades

4.4.7 Concluding remarks

As seen from the figures, results from de Sousa et al and AquaSim compare very well. Note that AquaSim results are sensitive to Raleigh damping stiffness.

4.5 Simplified representation of turbines in AquaSim

In analysis there is always a balance between the complexity of model and analysis time. In this section, a simplified representation of a wind turbine in AquaSim is presented. Right-click the node representing the turbine, and select Nodes > Pointload > Create new, as shown in Figure 13.

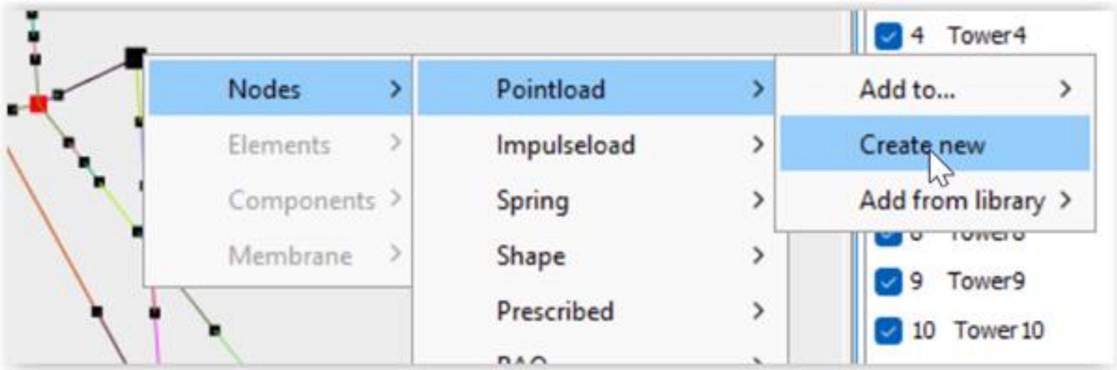

Figure 13 Pointload has been chosen. Here one may also insert a spring to the node

The dialogue-box for Pointloads appears, as shown in Figure 14. From the Load type-section, one can choose between several types of Pointloads.

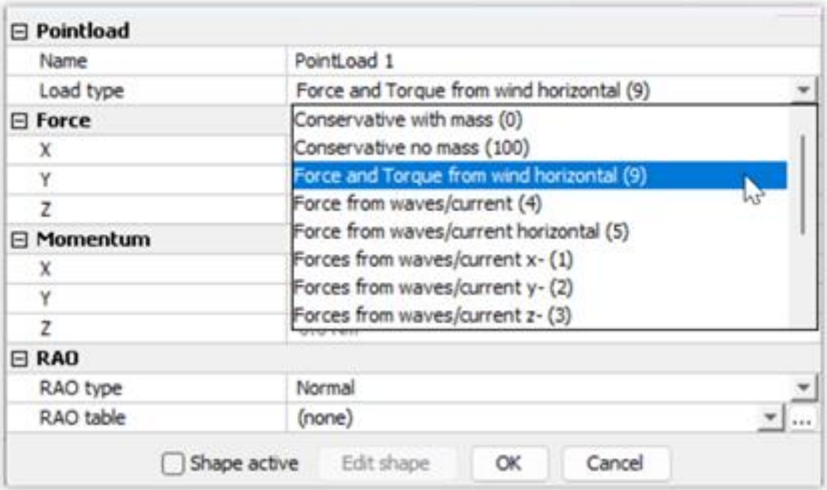

Figure 14 Different potions for Pointload types

Select *Force and Torque from wind horizontal*. In that case drag force and torque is proportional with wind load squared. The mass of the turbine may be applied as a spring of type *Mass*. This is shown in Figure 15 and Figure 16.

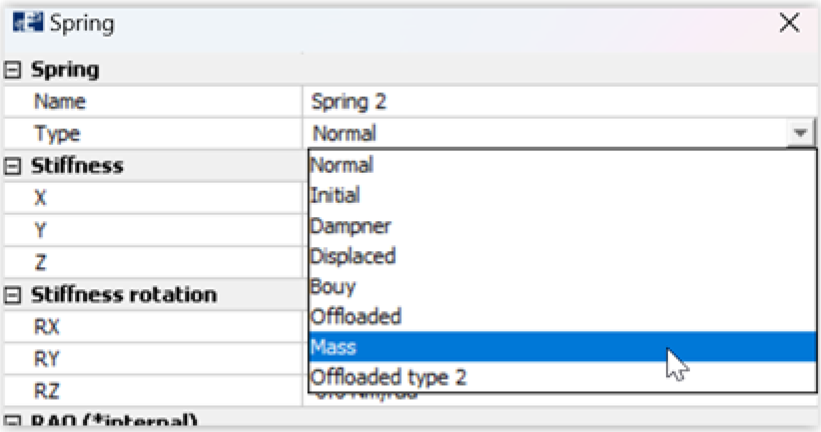

Figure 15 Choose Mass

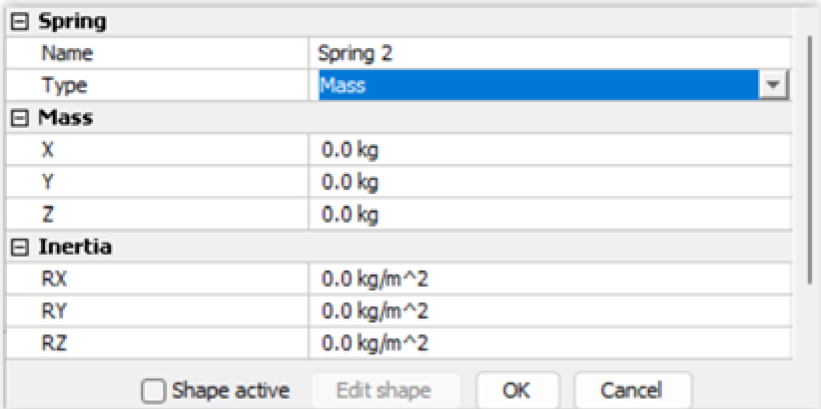

Figure 16 Node type Mass

As seen from Figure 16 one may apply both mass and inertia.

4.6 Damping

Doing comparison of analyzed turbine response to input data the importance of damping becomes evident. This section gives some input on that.

There are several ways to apply damping in AquaSim, each component can have Raleigh damping as shown in Figure 17.

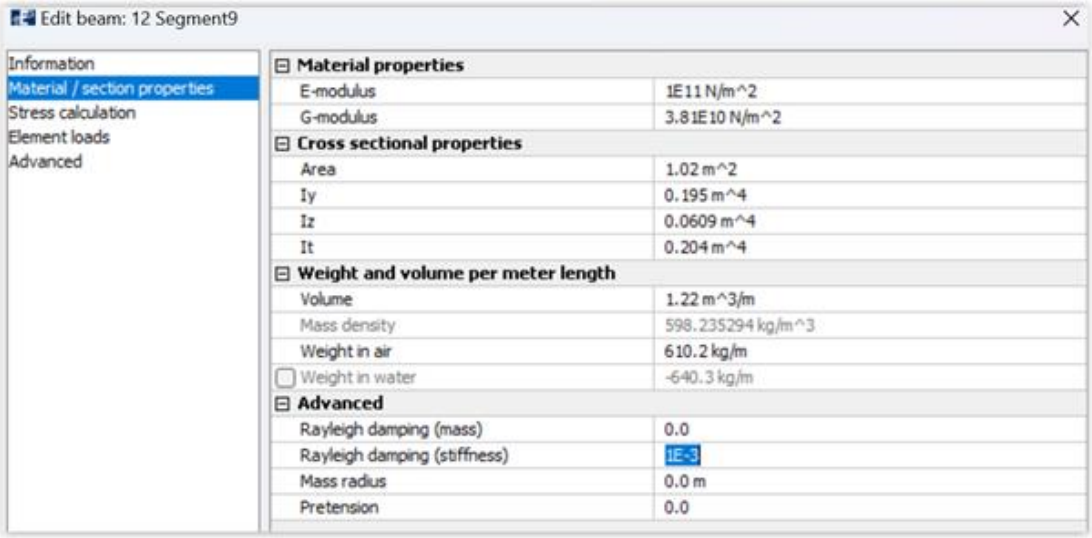

Figure 17 Raleigh damping

Damping can also be introduced through the Newmark-Beta method. In AquaSim, this type of damping is introduced through the Advanced section in the Export menu, as seen in Figure 18. This type of damping is then applied for the whole system.

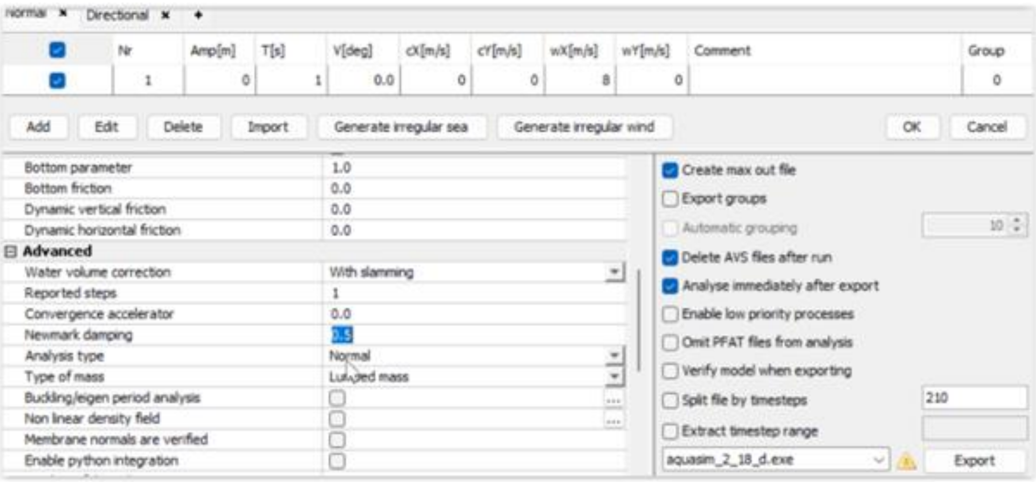

Figure 18 Damping in the Newmark-Beta methodology

A third way of introducing damping, is on the turbine blades through the *Blade tip damping* found in the *Advanced* section of the Generate turbine-tool.

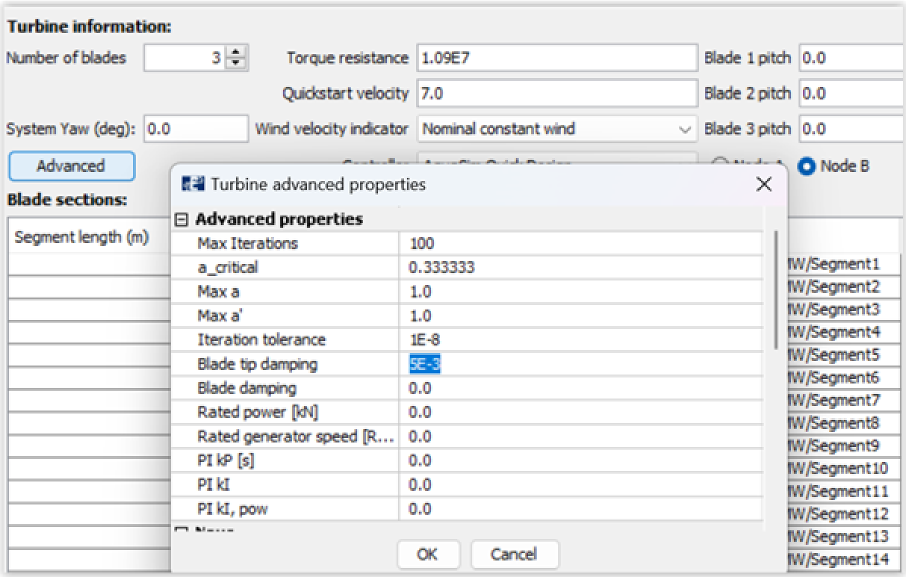

Figure 19 Blade tip damping from Advanced section on Generate turbine-tool

Damping is introduced to turbine blades through the factor *Blade tip damping* as seen in Figure 19 at the tip. Then it scales quadratically radius relative to the tip radius. This type of damping acts in the axial direction of the blade, and is increasing outwards on the turbine blade.

As turbine blades have a high velocity when spinning, damping will have a high impact. Some damping is real, and some damping is necessary for the convergence of analysis at rough conditions. Hence the following scheme could be used to obtain good analysis results in cases there damping is needed to avoid numerical issues:

- Apply as little as possible damping in the validation analysis to compare to data.
- Increase damping and reduce torque resistance (generator resistance)
- Insert a torque moment giving leading to the same torque in total.

Now we have a validated turbine and is ready to analyze systems with this turbine included.

4.7 Coupled analysis including turbines

Turbines may be installed on floating systems, jackets or on monopiles, and of course fixed on land. Some examples are shown in the figures below.

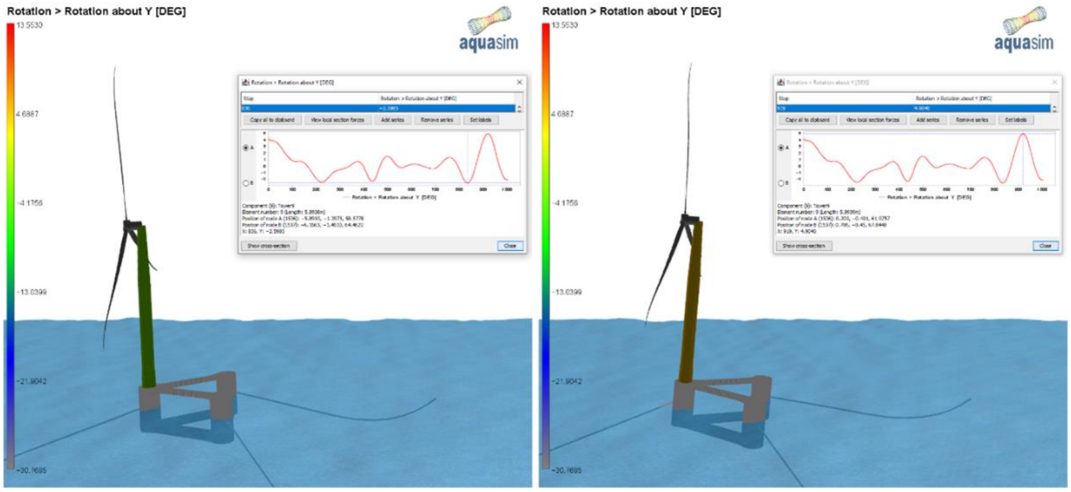

Figure 20 Turbine on floating system

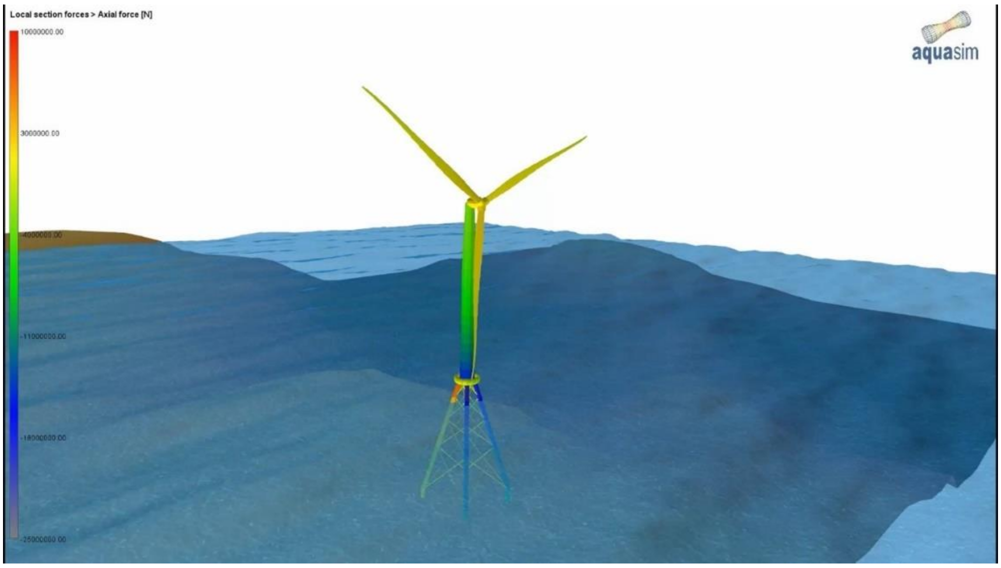

Figure 21 Turbine on bottom fixed installation

With AquaSim you may include environmental loads in combinations. The possibilities are:

- Waves: regular or irregular through spectra.
- Current: constant or varying with depth or time.
- Wind: constant or irregular through spectra.

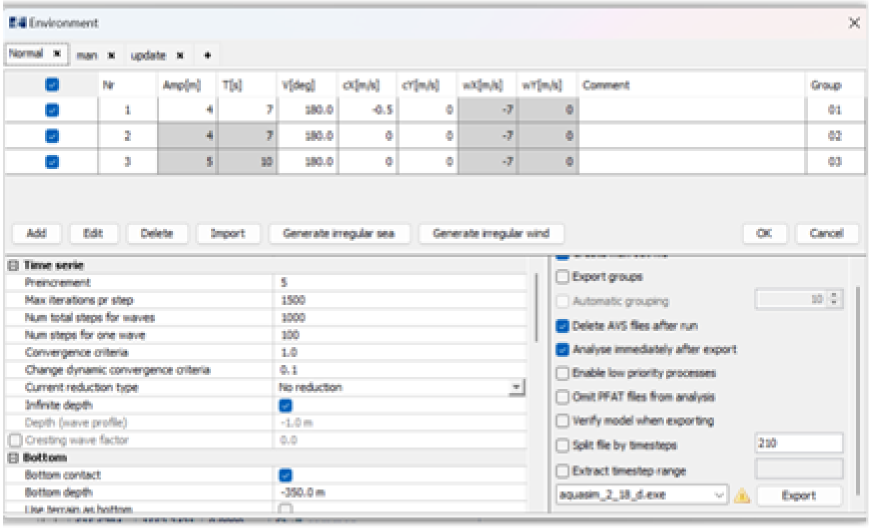

Figure 22 Environmental conditions for analysis

If one of the load combinations is double clicked on may choose from regular og irregular waves from predefined spectra, or irregular waves may be tailor made in any way as shown in Figure 23.

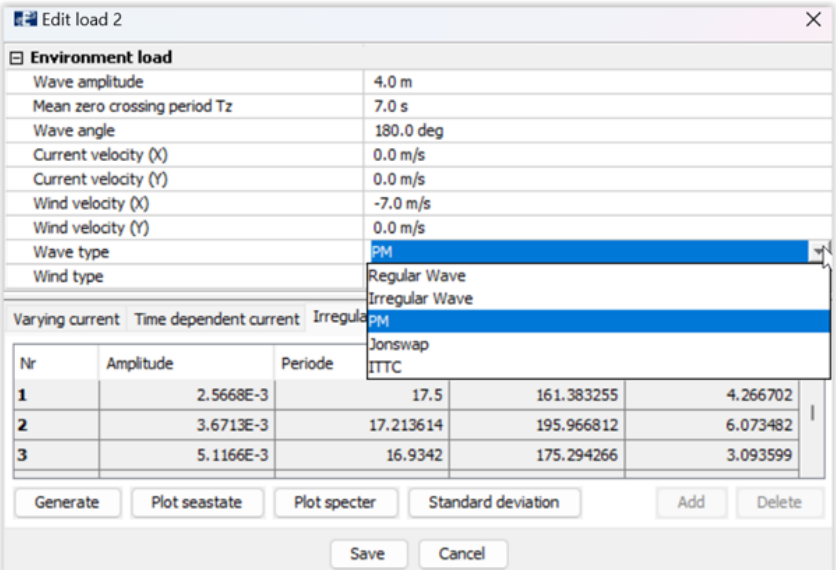

Figure 23 Available types of waves: regular or irregular

Having generated a wave spectrum, it can be plotted as shown in Figure 24 and Figure 25.

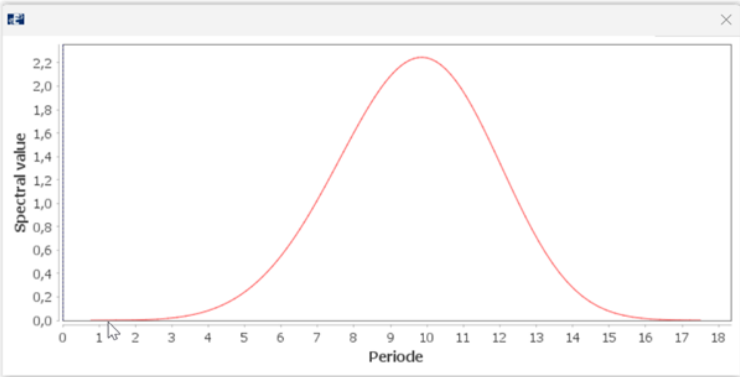

Figure 24 Spectrum

The wave height time series is shown in Figure 25.

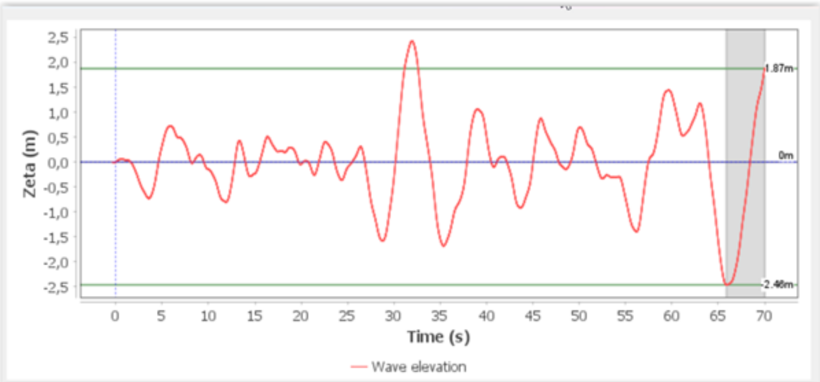

Figure 25 Tiem series of wave spectrum

As for waves, irregular wind can also be applied. The wind spectrum NORSOK is shown in Figure 26 and Figure 27.

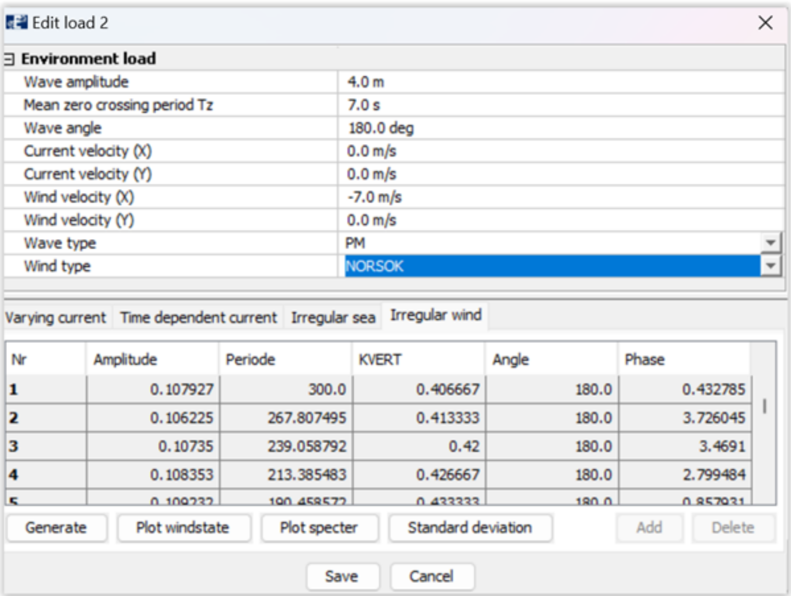

Figure 26 Type of wind: regular or wind spectrum

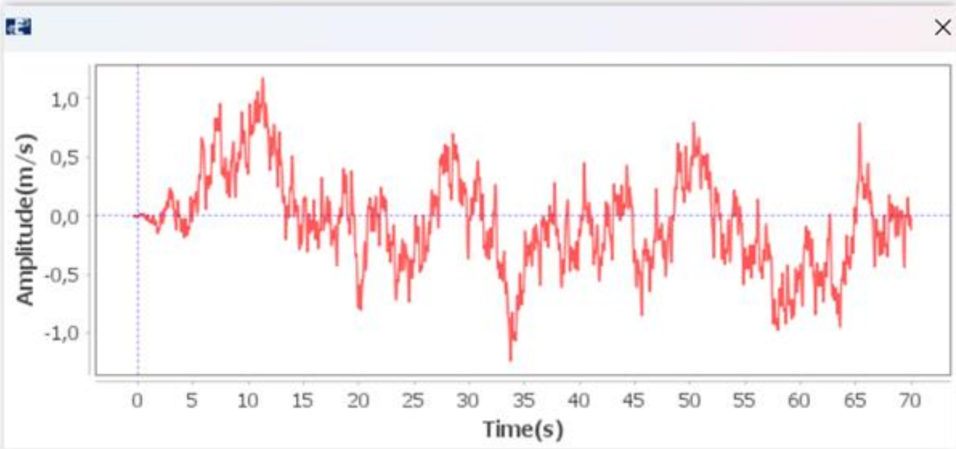

Figure 27 Time series of wind gust

As seen from Figure 26 one may choose to plot, wind state as a time series, the spectrum as well as the standard deviation. The time series of the wind gust is shown in Figure 27. Wind spectra and wind gust is further discussed in (Aquastructures, 2021). Analysis can be run in parallel for the desired combination of wind, currents, waves combined with the appropriate corresponding turbine parameters.

Depending on the modelled system – whether it is moored or fixed – some aspects may be more linear. Specially for bottom fixed, but there are natural periods in tower and such so it could be wise have a thorough nonlinear assessment also for this case, in particular considering that natural periods in the system may be very important and they will be nonlinear.

5 References

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