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AquaSim, Irregular wave calculation and validation

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Revision 2

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Summary:

AquaSim is an analysis program based on time domain analysis. Time series of waves are input to the program. The waves run by AquaSim can be a number of wave components where each wave component can be or made by user to derive at any combination of waves the structure is exposed to.

This report describes the parameters in AquaSim applied to generate irregular waves and its theory.

Revision 2:
In this revision, validation of ITTC and wave period cut off is evaluated for the latest version of AquaSim.

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Content

1 Introduction

AquaSim is an analysis program based on time domain analysis. Time series of waves are input to the program. The waves run by AquaSim can be a number of wave components where each wave component can be or made by user to derive at any combination of waves the structure is exposed to.

1.1 Generating waves to AquaSim

In the AquaSim solver a general set of sinusoidal waves can be read. These may be defined to any wave combination by the user.

However, for user convenience is may be of interest to generate a set of waves in AquaBase for running to AquaSim. In this case the AquaSim wave generator is run through AquaBase, and wave components for introduction to AquaSim are established and entered the AquaSim solver for the analysis.

The succeeding section describe the parameters use in and theory.

2 Time series generated from wave spectra

Note if irregular seas, parameter 1 is H_s and parameter to is T_z (mean zero crossing period of the spectrum). #WAVES shows the discretization of the spectrum. The random number seed determines the initial phase of each wave components.

2.1 Wave energy distribution to wave periods

Assuming infinite depth, linear Airy wave theory is applied where the wave potential can be expressed as:

$$\phi_1 = \frac{g\zeta_A}{\omega_0} cos(\omega t - kx \cdot cos\beta - ky \cdot sin\beta)$$

Equation 1

for a sinusoidal wave, where ζ_A is the wave elevation amplitude, ω_0 is the wave frequency of encounter, k is the wave number, g is the acceleration due to gravity and β is the wave heading relative to the global x- axis. ϕ_1 is the wave potential due to the incident waves (see e.g. (Faltinsen, 1990)). The wave elevation is then described as:

$$\zeta_A \cdot sin(\omega t - kx \cdot cos\beta - ky \cdot sin\beta)$$

Equation 2

Irregular waves at defined by a spectrum where the energy is divided over several wave components. In this case the wave elevation is described as:

$$\zeta = \sum_{n=1}^{N} A_n \, sin(\omega_{0n} t - k_n x \cdot cos\beta_n - k_n y \cdot sin\beta_n - \varepsilon_n)$$

Equation 3

where ε_n is a random number equally distributed between 0 and 2π. A_n is the amplitude of the n'th wave component found as:

$$\frac{1}{2}A_n^2 = S(\omega)\Delta\omega$$

Equation 4

where $S(\omega)$ is the spectral value. Wave spectra have characteristic values describing the how $S(\omega)$ varies with ω. An example of this can be:

$$S(\omega) = K_1\omega^{-5}exp(-K_2\omega^{-4})$$

Equation 5

where

$K_1 = \frac{\omega_z^4 H_s^2}{4\pi}$ and $K_2 = \frac{\omega_z^4}{\pi}$

Equation 6

which corresponds to the Pierson-Moscowitz spectrum. This is one of the spectra which can be applied in the AquaSim wave generator.
Most commonly, the parameter H_s is defined as:

$$H_s = 4\sqrt{m_0} = 4\sigma$$

Equation 7

where

$$\int_0^\infty S(\omega)d\omega = \sigma^2$$

Equation 8

where σ is the standard deviation of the process. Combining Equation 4 and Equation 8 it is seen that for all irregular wave patterns which are composed by a sum of sinusoidal waves, the standard deviation of the times series consisting og N wave components may be found as:

$$\sigma^2 = \sum_{i=1}^{N}\frac{A_i^2}{2}$$

Equation 9

introducing this to Equation 7 means H_s is found "backwards". This can be used as a self-check ensuring that any manually built wave introduced to AquaSim have the desired combined H_s.

Using the wave generator in AquaSim to generate wave, one shall not only introduce H_s but also T_z or $\omega_z = 2\pi/T_z$. This period is the mean zero crossing period of the spectrum defined as:

$$T_1 = 2\pi \cdot m_0/m_1$$

Equation 10

where

$$m_k = \int_0^\infty \omega^k S(\omega) d\omega$$

Equation 11

2.2 Spectra formulations

There exist a range of spectra. Also there exist a range of formulations to the parameters H_s and T_z. Following (Faltinsen, 1990) define:

$$T_2 = 2\pi\sqrt{(m_0/m_2)}$$

Equation 12

is another defined mean zero crossing period often associated with the T_z input value to the PM spectrum formulation. What is "correct" values here depends on how the spectral formulations are defined in consistency with the definition of the mean zero crossing period, T_z. Both, T_1 and T_2 are definitions of T_z. T_1 is most common together with JONSWAP spectra whereas T_2 is most common together with PM spectra formulations. This is valid also here. Another commonly used parameter in connection with spectra is, T_p which is the peak period of the wave spectrum. For a PM spectrum this period is approximately $1.41 \cdot T_2 = 1.30 \cdot T_1$.

2.2.1 PM spectrum basic formulation

The basic PM spectrum in AquaSim is:

$$S(\omega) = \frac{A}{\omega^5} exp\left(\frac{-B}{\omega^4}\right), \;\; B = \frac{\omega_0^4}{\pi}, \;\; A = B \cdot \frac{H_s^2}{4}, \;\; \omega_0 = \frac{2\pi}{T_z}$$

Equation 13

where T_z is the average zero crossing period of the spectrum. Often this is for this PM spectrum formulation the T_2 in Equation 12. This is the basic PM spectrum formulation in AquaSim.

2.2.2 JONSWAP spectrum formulation

The JONSWAP wave spectrum in AquaSim is the one in Equation 2.30 in (Faltinsen, 1990).

$$S(\omega) = 155\frac{H_s^2}{T_z^4\omega^5} exp\left(\left(\frac{-944}{T_z^4\omega^4}\right) \cdot \gamma^Y\right), \;\; Y = exp\left(-\left(\frac{0.191\omega T_z - 1}{\sqrt{2}\sigma}\right)^2\right)$$

Equation 14

$\sigma = 0.07 \; for \; \omega \leq 5.24/T_z$

$\sigma = 0.09 \; for \; \omega > 5.24/T_z$

$\gamma = 3.3$ as default value. $\gamma = 3.3$ corresponds to JONSWAP spectrum in (Faltinsen, 1990). T_z is T_1 as in Equation 10.

2.2.3 ITTC spectrum formulation

The ITTC spectrum formulation according to (Faltinsen, 1990), Eq. 2.24 is given in Equation 15. In this case T_z is T_1 as shown in Equation 10.

$$S(\omega) = H_s^2 T_z \frac{0.11}{2\pi}\left(\frac{\omega T_z}{2\pi}\right)^{-5} \cdot exp\left(-0.44\left(\frac{\omega T_z}{2\pi}\right)^{-4}\right)$$

Equation 15

2.2.4 Wave period cut off

All the above spectral formulations have a decay in energy following ω^{-5} as the wave frequency gets high. The integral of this decays with ω^{-4}. That means the wave amplitudes of the generated waves will decrease with ω^{-2} if one makes the $\Delta\omega$ successively longer as ω increase. This means load terms proportional with the wave acceleration will have a challenge formulation wise. For 2^{nd} order or other non-linear terms this may represent a large challenge. In the spectral wave generation in AquaBase, a cut off value is automatically chosen. The user should however be aware of effects around this.

2.3 Randomization of wave periods

Figure 1 shows indicatively how the wave energy in a spectrum may be represented by a set of waves. If randomization of wave period is not chosen, the period of the generated wave will be the middle period in the interval the wave component is set to represent. This is indicated in Figure 2.

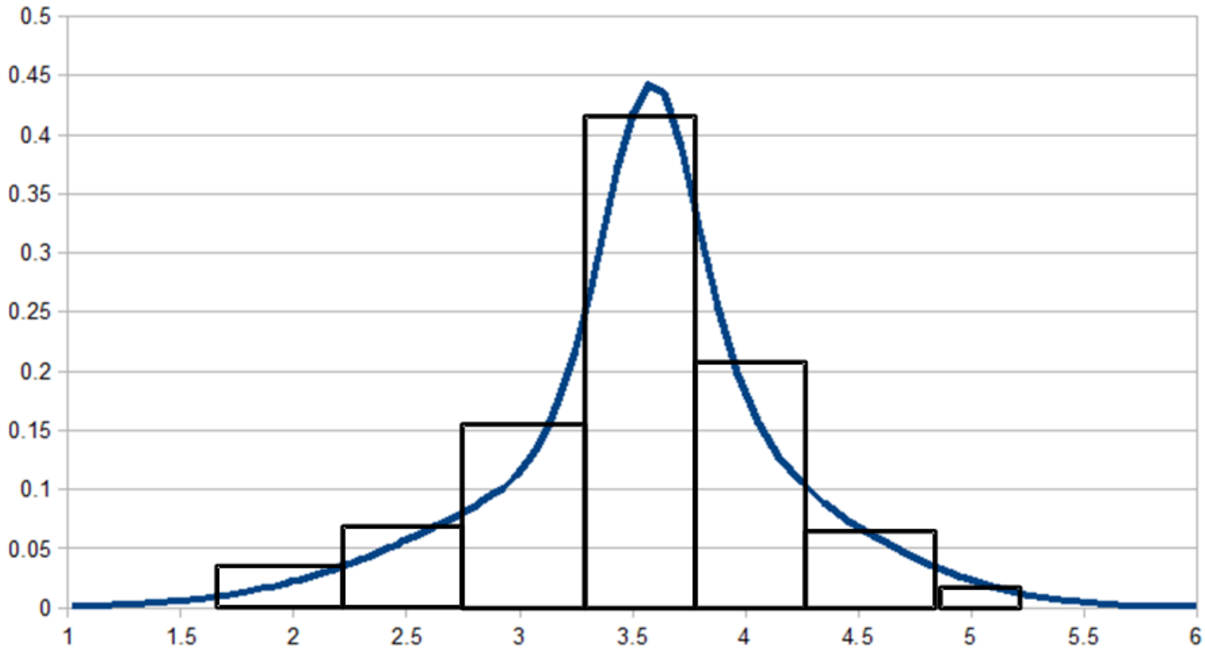

Figure 1 Illustration of how spectrum is represented by a set of waves

In Figure 1, the red lines are placed in the centre of the interval. This is where the period of the wave component is in case wave periods are not randomized. If periods are randomized the wave period is places randomly along the blue line for the given period range. This is seen by the blue lines in Figure 2.

Each interval holds an independent random number determined by the random seed.
Note that if randomization of waves is not chosen the patters of the generated waves have a tendency to self-repeat. If randomization is chosen self-repeating takes time much later.
If randomization is not chosen, this may often lead to smaller wave peaks than if randomization is chosen. The maximum wave at a specific point (x_0, y_0) can be lower in a generated time interval than the normal $1.9 \cdot H_s$ which is a commonly used value when applying a design wave.

2.4 Short crested waves

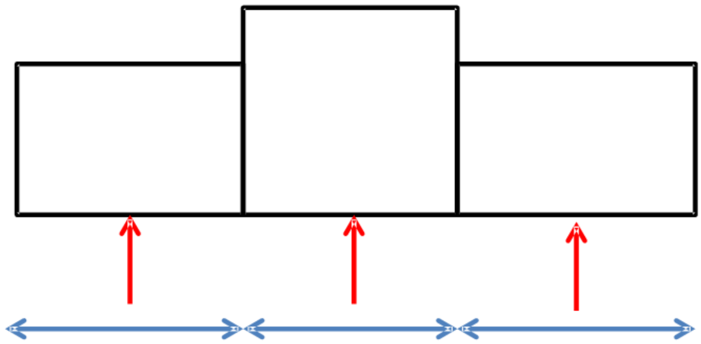

Figure 2 Mid point in interval indicated by red lines. Interval is the blue lines.

In case of short crested waves each wave component is associated with a randomization with respect to the wave direction of the wave component. The directionality of the wave is then given by:

$$\beta_i = cos^n(\phi_i)$$

Equation 16

Where n is given as input by user. 2 is a commonly used value (see e.g. (Faltinsen, 1990)). $\phi_i = [-\pi/2\,, \pi/2]$ are equally randomly distributed in the interval $[-\pi/2\,, \pi/2]$. i is the index of the i^{th} wave component in the time series representing the irregular waves. The the direction angle β is added to the wave direction given as input to the seastate.

Short crested waves may be introduced in two ways either N number of waves components where each component have a randomly drawn angle in the interval $[-\pi/2\,, \pi/2]$ weight by the appropriate $cos^n(\phi)$, or it may be run in symmetry mode such that to each wave component with angle within $[0\,, \pi/2]$ there is a corresponding at angle $[-\pi/2\,, 0]$ with the same phase and transposed angle. In this case $2N$ waves are generated.

3 Validation

3.1 Spectral distribution

Figure 3 Shows the spectral density of the 3 main spectra:

- JONSWAP which is the JONSWAP spectra according to Equation 14.
- PM which is the PM spectra given in Equation 13.
- ITTC which is the ITTC version of the PM spectrum given in Equation 15.

In addition the spectrum originating by letting the peak factor being 3.3 in the original JONSWAP spectrum (Faltinsen, 1990) be 1 is shown in Figure 3. All spectra are scaled as shown by the axis. Note that in the PM spectrum offered by AquaSim, T_z as input is T_2 (Equation 12) in case the PM spectrum is used whereas it is T_1 (Equation 10) in case the other spectra is applied.

As seen from Figure 3, PM, ITTC and JONSWAP_1.0 are basically equal. In order to validate the spectra seen in Figure 3, the spectral values can be calculated back from the established waves components following Equation 4.

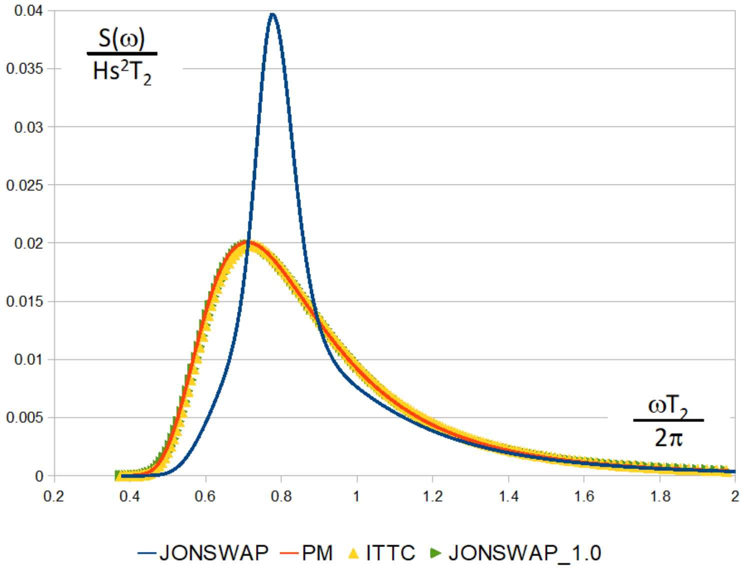

Figure 3 Spectra

In order to validate that the spectral distribution is calculated correctly by AquaSim the distribution seen in Figure 3 is found reversely based on the waves generated by the spectral routines.

3.2 Validation of Hs

The first validation is a simple case of checking that H_s is correct calculated from the wave components. Combine Equation 7 and Equation 9:

$$H_s = 4\sqrt{m_0} = 4\sqrt{\sigma^2} = 4\sqrt{\sum_{i=1}^{N} \frac{A_i^2}{2}}$$

Equation 17

Using Equation 17 to calculate H_s gives results as seen in Table 1. It shows that correct H_s is calculated from the spectra.

Table 1 Hs and Tz reversely calculated from the different spectra

Spectra	Hs [m]	Tz(T1) [s]	Tz(T2) [s]	T1/T2	Cutt off w [2p/s]
JONSWAP	2.000	3.000	2.800	1.071	18.850
PM	2.000	3.036	2.800	1.085	20.226
ITTC	2.000	3.005	2.771	1.084	18.850

3.2.1 Validation of spectral distribution

In order to validate the waves calculated based on the spectral distribution, starting out with Equation 4 it is seen that:

$$S(\omega_i) = \frac{1}{2} \cdot \frac{A_i^2}{\Delta\omega_i}$$

Equation 18

Figure 4 shows the spectra reversely calculated from the output waves by applying Equation 18. As seen by comparing Figure 4 to Figure 3, results compare well.

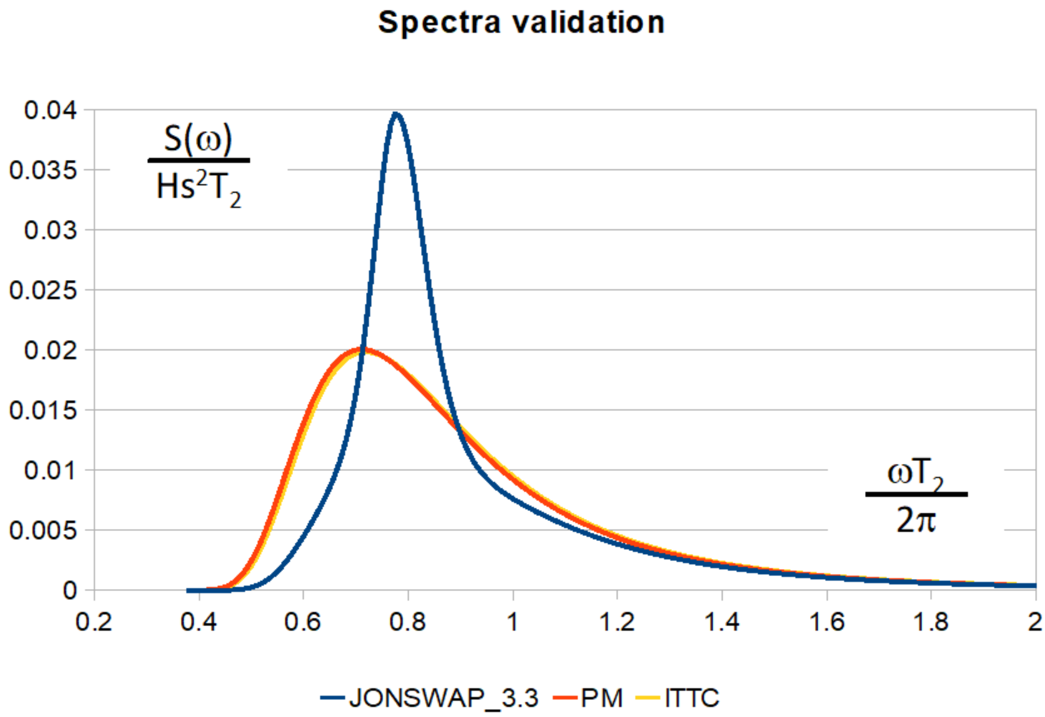

Figure 4 Spectral distribution reversely calculated from output waves

3.2.2 Validation of average zero crossing period

The average zero crossing period, $T_z = T_1$ is found by Equation 10. By introducing Equation 11 to Equation 10 and carrying out the integration numerically, T_1 is found as:

$$T_1 = 2\pi \cdot \frac{m_0}{m_1} = 2\pi \cdot \frac{\int_0^\infty S(\omega)d\omega}{\int_0^\infty \omega S(\omega)\,d\omega} = 2\pi \cdot \frac{\sum_{i=1}^N S(\omega_i)\Delta\omega_i}{\sum_{i=1}^N \omega_i S(\omega_i)\Delta\omega_i} = 2\pi \cdot \frac{\sum_{i=1}^N A_i^2}{\sum_{i=1}^N \omega_i A_i^2}$$

Equation 19

In the same way, T_2 is found as:

$$T_2 = 2\pi \cdot \sqrt{\frac{m_0}{m_2}} = 2\pi \cdot \sqrt{\frac{\int_0^\infty S(\omega)d\omega}{\int_0^\infty \omega^2 S(\omega)d\omega}} = 2\pi \cdot \sqrt{\frac{\sum_{i=1}^N S(\omega_i)\Delta\omega_i}{\sum_{i=1}^N \omega_i^2 S(\omega_i)\Delta\omega_i}} = 2\pi \cdot \sqrt{\frac{\sum_{i=1}^N A_i^2}{\sum_{i=1}^N \omega_i^2 A_i^2}}$$

Equation 20

Inserting the resulting waves from the wave calculation into Equation 19 and Equation 20 respectively, T_1 and T_2 is found as shown in Table 1. As seen these values deviates slightly from the nominal values. This is due to period cut-off (cut-off of low periods, high frequency).

3.3 Period randomization

Figure 5 shows a case where the spectrum has bee approximated 20 waves. Randomization (Random_1 and Random_2) have been compared to no randomization (No_random).

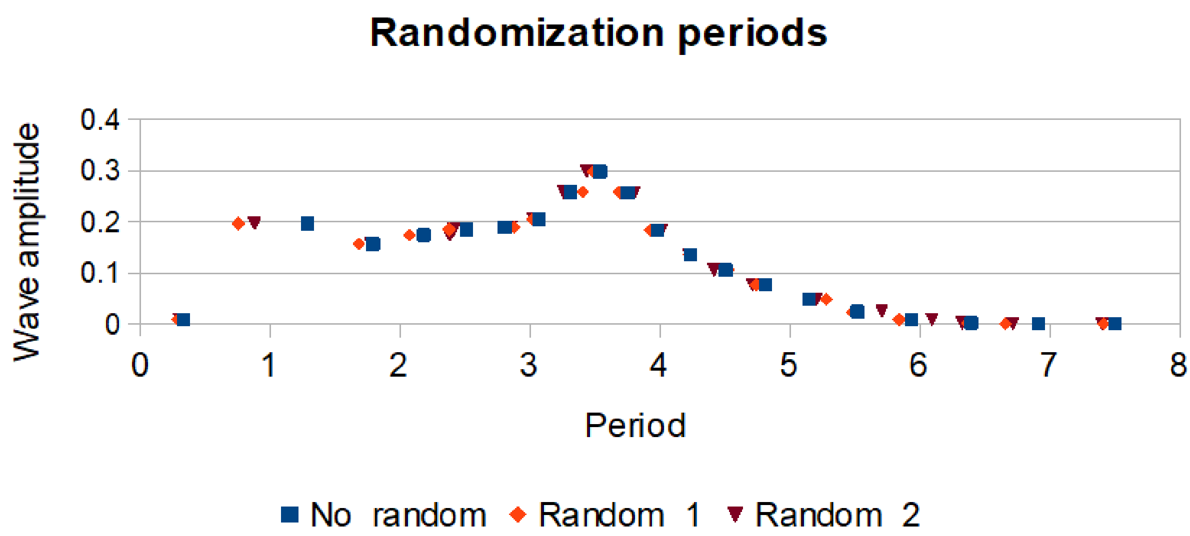

Figure 5 Periods for analysis, one case without randomization of periods as two cases with randomization with different seeds.

3.4 Short crested waves

The probability density function for a variation of $\beta = cos^n(\phi)$ where $n = 1,2$ and 20 respectively is shown in Figure 6. $cos20$ angle, ϕ, means $\beta = cos^{20}(\phi)$ and similar.

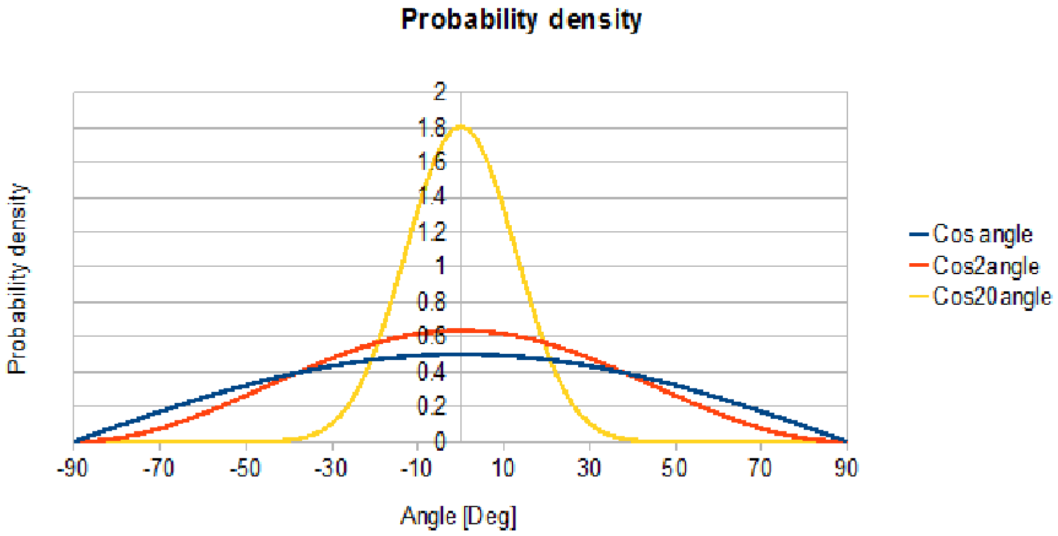

Figure 6 Probability density function

Generation of short crested waves are tested by running a case with 10,000 individual wave components and making a probability density plot on how the realizations are distributed. This is shown in Figure 7. Due to the randomization of making the angle of the actual wave component this realization is not as smooth as the distributed plot.

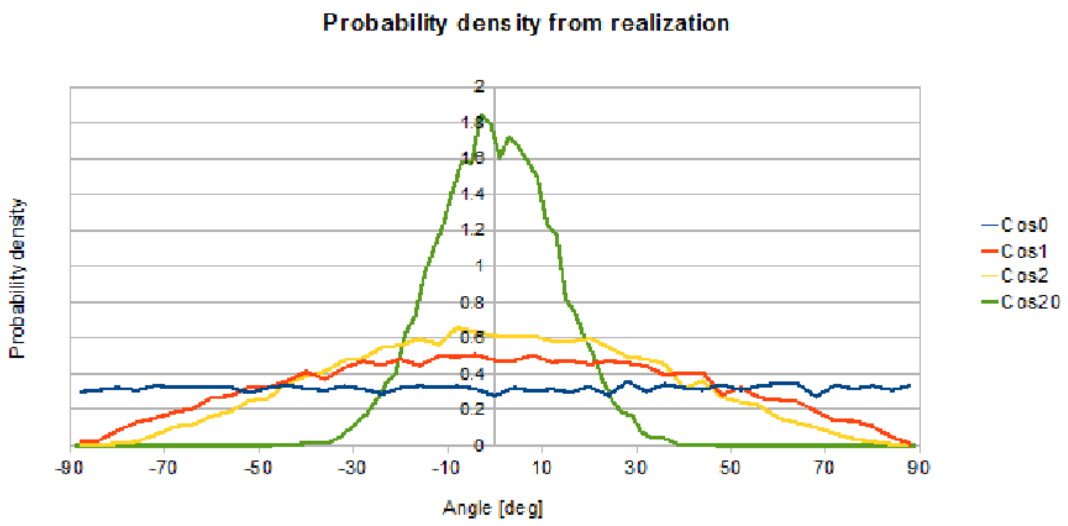

Figure 7 Distribution of realizations, probability.

In Figure 7, $cos0$ means that the distribution is $\beta = cos^0(\phi)$ which means even in the interval. As seen from the figure is correspond well. Note that in Figure 7 there is not full symmetry. If symmetry is desired, the wave generation of short crested waves should be carried out in symmetry mode.

3.4.1 Short crested waves, symmetry mode

In this case the wave generation and possibilities for $\beta = cos^N(\phi)$ $\beta = \cos^N \phi$ are the same as with no symmetry, but twice as many waves are generated and to each wave with an angle β, a wave with angle $-\beta$ also exist with the same phase angle. In this case twice as many wave components are generated. Figure 8 shows short crested waves run in symmetry mode where results have been beck tracked to compare to the probability distribution. As seen from the figure, results compare well.

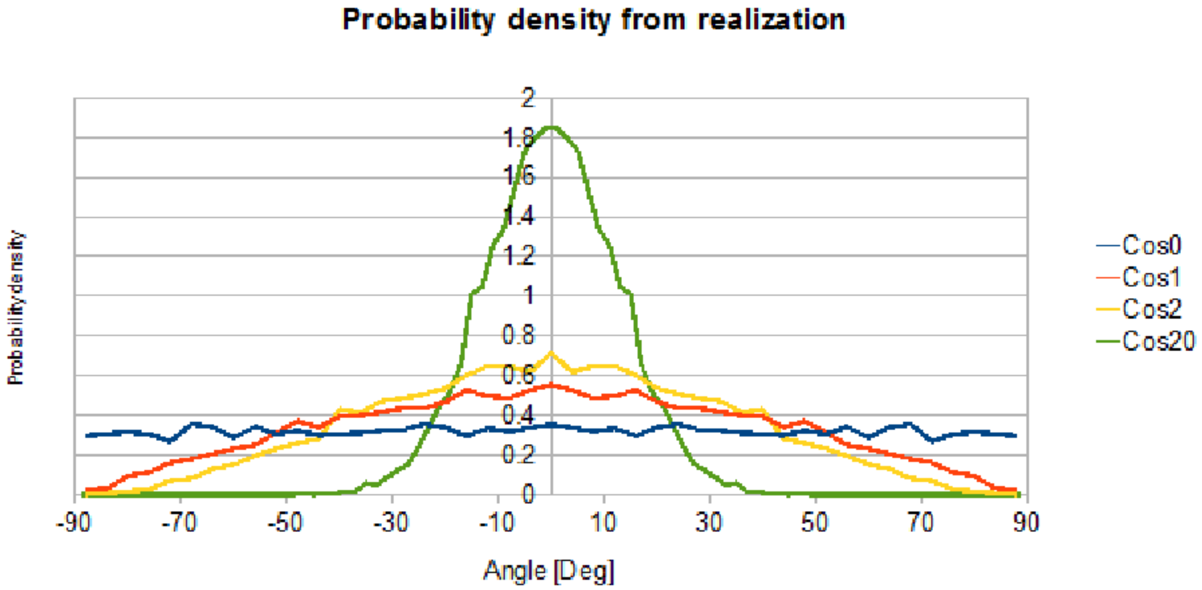

Figure 8 Distribution of realizations, probability, symmetry mode.

3.5 Time series

The AquaSim wave generator reports back the maximum positive and negative wave amplitude and maximum wave height in the interval.

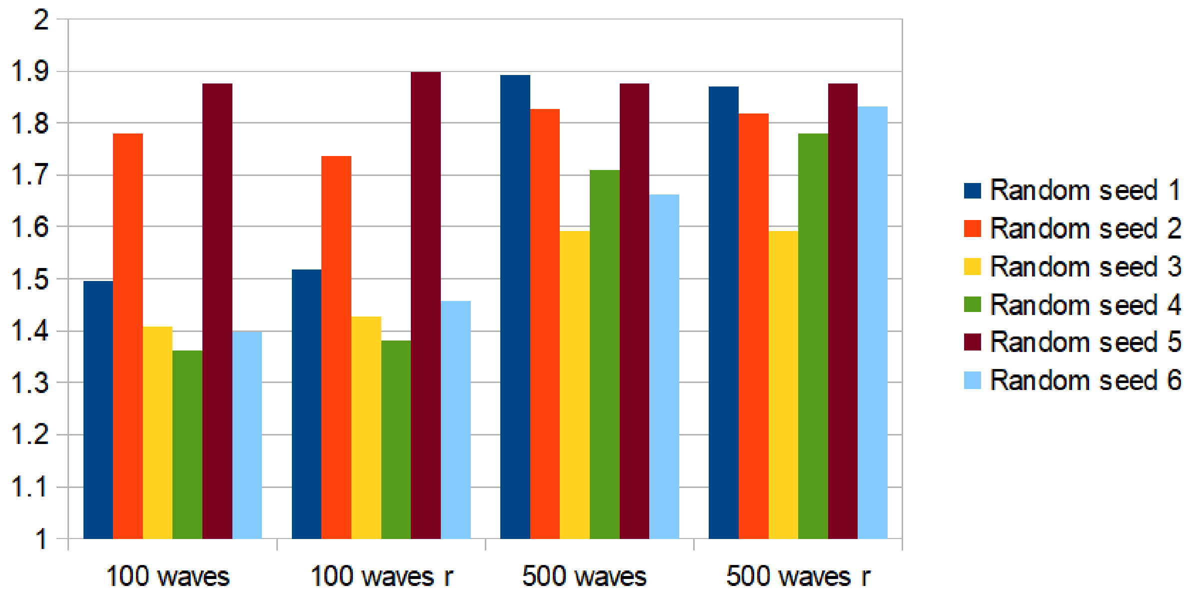

Figure 9 Max wave height in realization relative to Hs.

In Figure 9, 100 waves means 100 wave cycles have been run. The letter r means periods are randomized. As seen from the figure, the maximum relative wave height may be significantly lower than 1.9 times H_s which is rule demand in case a regular wave is used for analysis. This means one may risk having the largest wave in a realization lower than the standards design waves. This can be checked by checking #time.txt output file. At the end of this file max and min values are given.

4 References

Faltinsen, O. (1990). *Sea Loads on Ships and Offshore Structures.* Cambridge University Press. ISBN 0-521-37285-2.