REPORT

ON THE ANALYSIS OF MOORED LARGE MASS FLOATING OJBECTS AND HOW TO CARRY OUT SUCH ANALYSIS WITH AQUASIM

REPORT NO. TR-FOU-2174-1

REVISION NO. 01

AQUASTRUCTURES

Date of first issue: 31 October 2013	Project No.: 2174
Approved by: Are Berstad, Project Manager	Organisational unit: Research and development
Client: Norges forskningsråd	Client ref.:

Summary:

Several analyses have been carried out validating the ability of AquaSim to calculate response of large mass moored vessels.
The hydrodynamic loads to the vessel is calculated integrated with motion and forces in the mooring system such that nonlinear response is accounted for. This document shows how forces from hydrodynamic loads are introduced to AquaSim. The document describes the options AquaSim users have to account for the load components which are of importance for response.

Relevant natural periods of systems and loads occurring should be carefully evaluated by the analyst to ensure that all relevant load-response-effects are considered.

The importance of velocity and mass of floating object, stiffness properties of moorings are shown both through analysis and analytical formulae. Analysis using AquaSim compares well with analytical formulae for cases where analytic formula can be established.

AquaSim shows good versatility.

Report No.: TR-FOU-2174-1	Subject Group:	
Report title: ANALYSIS OF MOORED LARGE VOLUME FLOATERS WITH AQUASIM		
Work carried out by: Are Johan Berstad		
Work verified by: Line Fludal Heimstad		
Date of this revision: 31. Okt 2013	Rev. No.: 1	Number of pages: 41

Indexing terms

Finite Element Analysis
Hydroelasticity
Flexible structures and systems
Sea loads

☐ No distribution without permission from the Client or responsible organisational unit

☐ Limited distribution within Aquastructures AS

☒ Unrestricted distribution

Table of Content ***Page***

1. INTRODUCTION

Method to find load to structure (RAO, Hydrodynamic theory, Morison)

2. LOADS AND RESPONSE TO LARGE MASS FLOATING OBJECTS

A typical barge is shown in Figure 1.

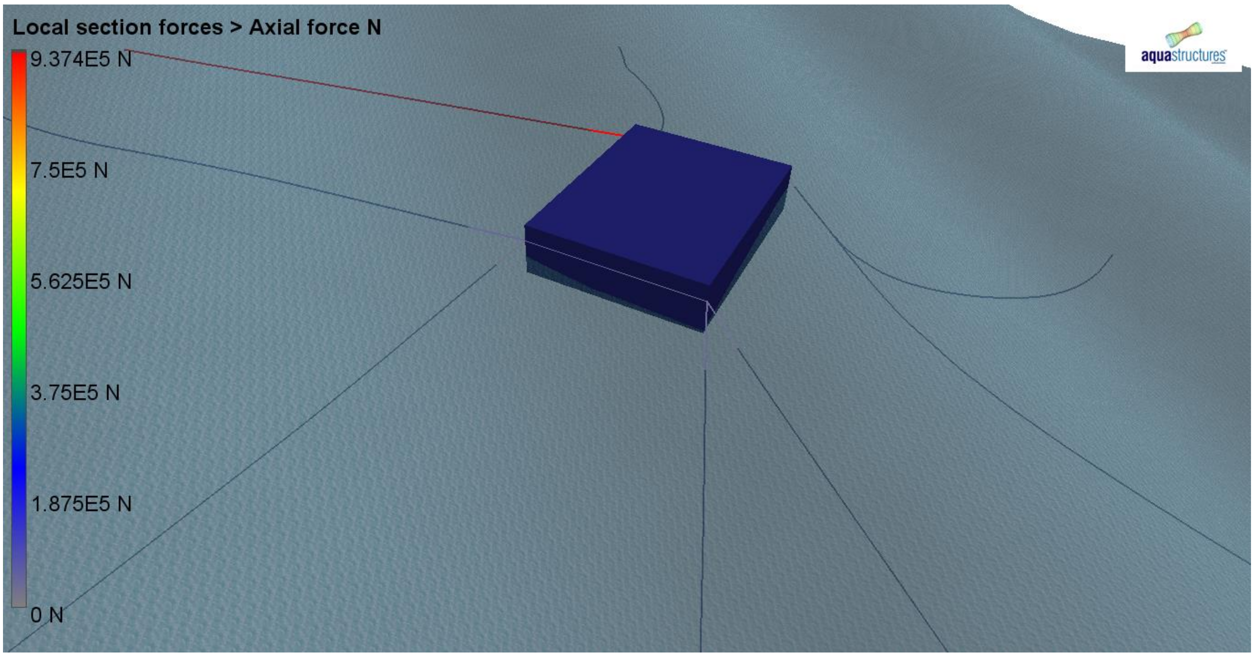

Figure 1 Floating barge with spread mooring

Analyzing barges and other floating objects there are several response aspects of interest. The floating object is exposed to loads from the sea in terms of waves and current. In air there is the wind also leading to forces. The mooring is keeping the floating object in place in the horizontal plane as shown in Figure 2. In the vertical plane, it is mainly the water plane stiffness holding the barge in place as seen in Figure 3.

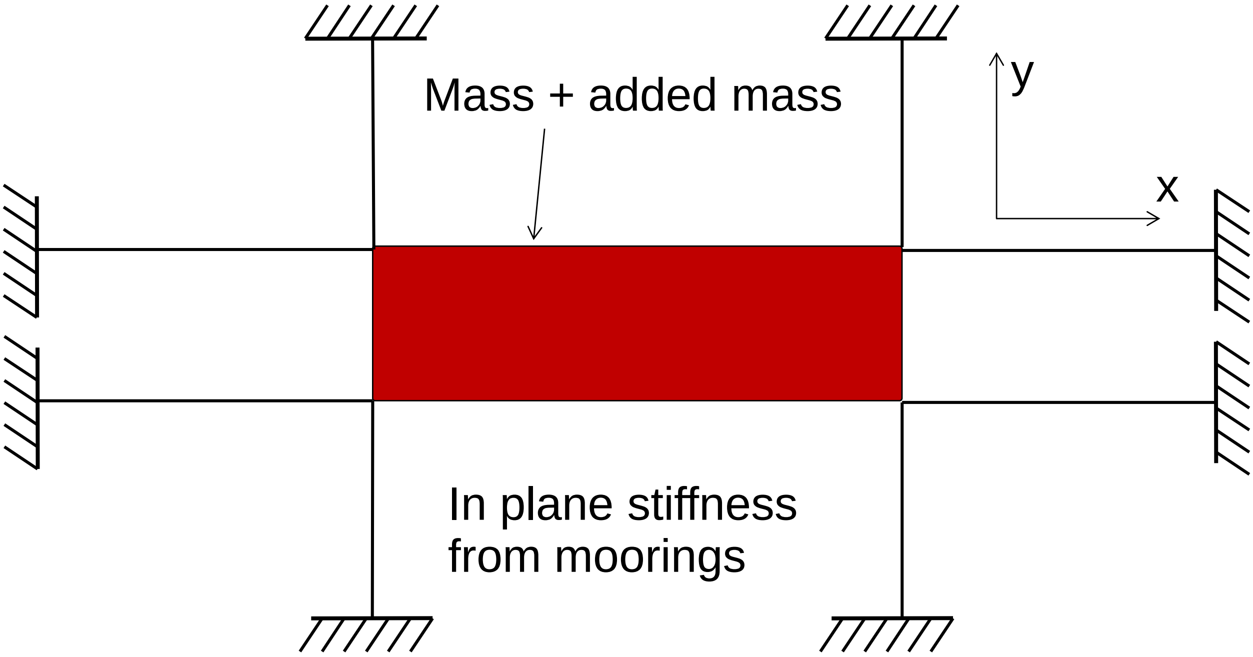

Figure 2 Moored barge seen from above

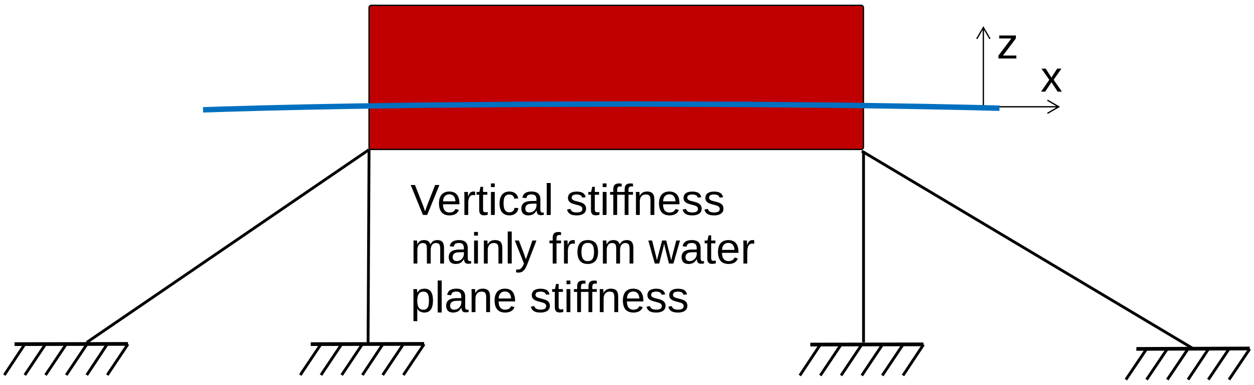

Figure 3 Moored barge seen from side

2.1.Hydrostatic forces acting on a body in water

Consider an object floating in water as shown in Figure 4.

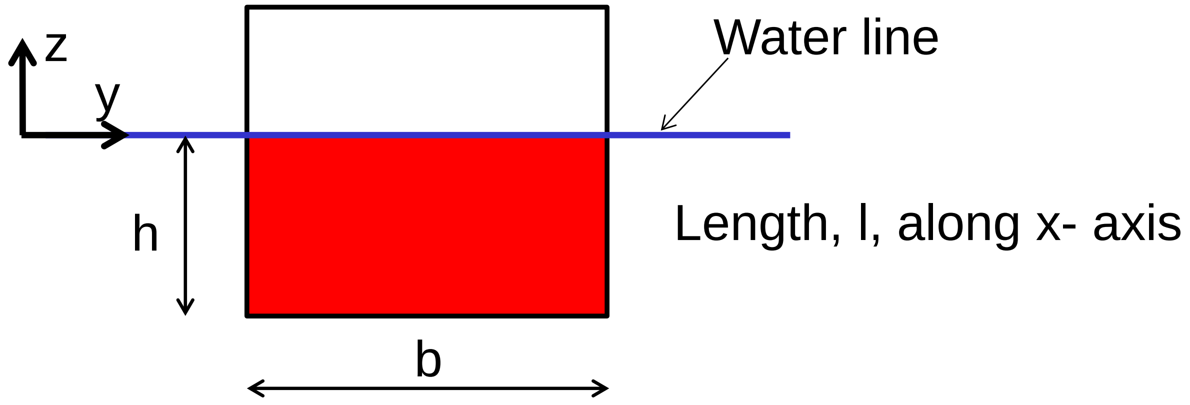

Figure 4 Rectangular vessel seen in the *y-z* plane.

2.1.1. Buoyancy

The forces acting from the water to the structure is the integral of the fluid pressure around the object. Define an orthonormal coordinate system where the x- axis is along the object in the horizontal plane, the z- axis is upwards with origin at the mean water line as shown in Figure 3. Hydrostatic pressure increase downwards in a fluid and the hydrostatic pressure at a given point in a fluid (see e.g. http://en.wikipedia.org/wiki/Fluid_statics) can be found as:

$$p = -\rho g z + p_{atm}$$

Equation 1

Where ρ is the density of water, g is gravity, z is the vertical location (origin at free surface and axis pointing upwards) p_{atm} is the atmospheric pressure in air at the free surface.

Assume the fluid is non-viscous. Then a force originating by fluid pressure will be directed normal to the surface. Introducing this to the case seen in Figure 4 it is seen that the net horizontal force is 0 due to symmetry and the net vertical force is

$$F = \rho g h b l$$

Equation 2

Where F is positive upwards and l is the length out of the plane seen in Figure 4 and h and b are as defined in the figure. Equation 1 can be rewritten to

$$F = \rho g V$$

Equation 3

Where V is the submerged volume. As seen this is in accordance with Archimedes principle (http://en.wikipedia.org/wiki/Archimedes%27_principle).

As seen from Figure 4 moving the body downwards will give a higher submerged volume and hence more buoyancy, pushing the vessel down a distance *-dz* means the force upwards will be

$$F = \rho g(h + dz)bl$$

Equation 4

Meaning the extra force upwards is

$$F = \rho gbldz$$

Equation 5

2.1.2. Water plane stiffness

Using the definition of spring stiffness according to Hookes law, $F = kdz$ (http://en.wikipedia.org/wiki/Hooke%27s_law) gives the "water plane stiffness" , k_w

$$k_w = \rho gbl = \rho gA_w$$

Equation 6

where A_w is the water plane area which is the area of the water line which is penetrated by the floating object.

2.1.3. Eigenperiod heave

Neglecting stiffness and mass from moorings, the eigenperiod for a floating body with a water plane area is

$$T_e = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m + m_a}{\rho gA_w}}$$

Equation 7

Where m_a is the added mass in heave.

2.1.4. Roll motion and stability

Doing a similar consideration for roll motion, which is rotation about the x- axis finds that restoring moment depends on the distance between the centre of buoyancy, B and how the centre of buoyancy moves when the floating object is rotated. This is shown in Figure 5.

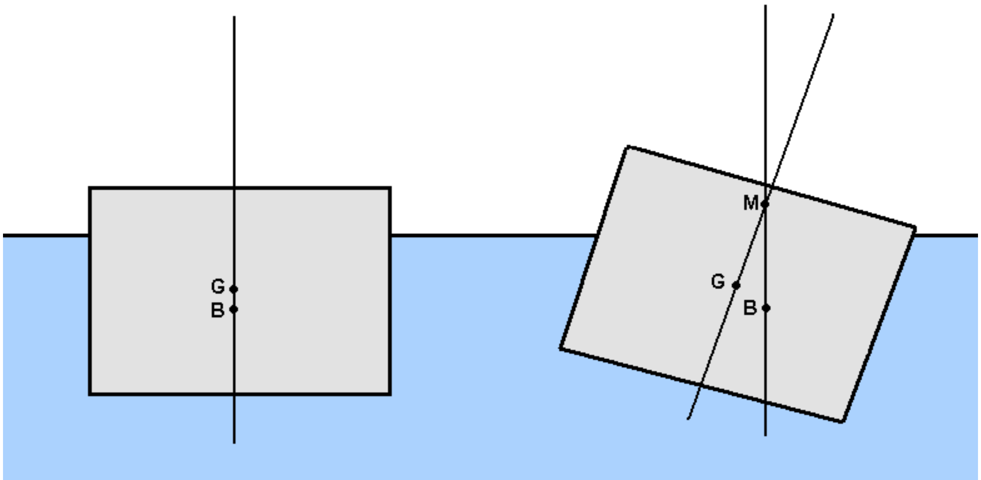

Figure 5

Analogue to water plane stiffness relative to z- motion there is a rotation stiffness for roll depending on GM where GM is defined at the vertical distance from the floating objects centre of gravity to the Metacentric height where

$$\mathrm{M} = \mathrm{B} + \frac{\mathrm{lb}^3}{12V} = B + \frac{I_x}{V}$$

Equation 8

where I_x is the area moment of inertia of the water plane area about the *x*- axis.

2.1.5. Eigenperiod roll

The eigenperiod, T_e is

$$T_{\mathrm{e}} = 2\pi\sqrt{\frac{\mathrm{m}}{k}} = 2\pi\sqrt{\frac{\rho \mathrm{V} \mathrm{r}^2 + \mathrm{m_a}}{\rho g V(GM)}}$$

Equation 9

Where r is the average mass radius of inertia (http://en.wikipedia.org/wiki/Radius_of_gyration)
Often the added mass inertia in roll may be smaller than the mass inertia of the vessel itself.
Neglecting the added mass, the natural period is

$$T_{\mathrm{e}} = 2\pi\sqrt{\frac{\mathrm{r}^2}{g(GM)}} = \frac{0.64r}{\sqrt{GM}}$$

Equation 10

Hydrostatically, the vessel is in equilibrium in the horizontal direction since the pressure acting on both sides of the vessel is similar. A moored vessel will find horizontal equilibrium where the horizontal components of the mooring forces cancel out each other.

Forces downwards by moorings leads to a larger submerged volume of the vessel giving an upward force equalizing the downward force by the moorings. In this case, the total buoyancy from the submerged volume equals the weight of the vessel plus the downward force of the moorings.

2.1.6. Pitch and yaw motions

In AquaSim pitch and yaw rotations are treated by the FE method of the program.

2.1.7. Input for calculation of hydrostatic properties in AquaSim

Table 1 shows the input parameters given to AquaSim for calculation of hydrostatic properties. Table 2 shows the hydrostatic parameters derived by calculation within AquaSim. Combined with mass and added mass one also have parameters for calculation of eigenperiods.

Table 1 Input parameters

Parameter	**Description**
Hull shape	A given number of points describing the hull
COG	Vertical centre of gravity structure, mass centre
Water line	Vertical location of water line
Mass radius	Mass radius of gyration

Table 2 Hydrostatic parameters calculated by AquaSim

Parameter	**Description**
COB	Vertical centre of buoyancy
Aw, kAw	Water plane area, water plane stiffness
Iw	Area moment of inertia, roll water plane
CM	Metacentric height

2.2. Hydrodynamic forces acting on a body in water

Waves are a time dependent change in the water elevation and also the pressure in the fluid. The pressure below the water surface is in this case normally parted to the static part and the dynamic part of the pressure where the dynamic part of the pressure is a perturbation of the average hydrostatic pressure.

2.2.1. Wave theory

Let the wave elevation be described by Airy waves (http://en.wikipedia.org/wiki/Airy_wave_theory). The water particles will then move in a circular pattern at infinite depth and an elliptic pattern in finite depth as shown in Figure 6.

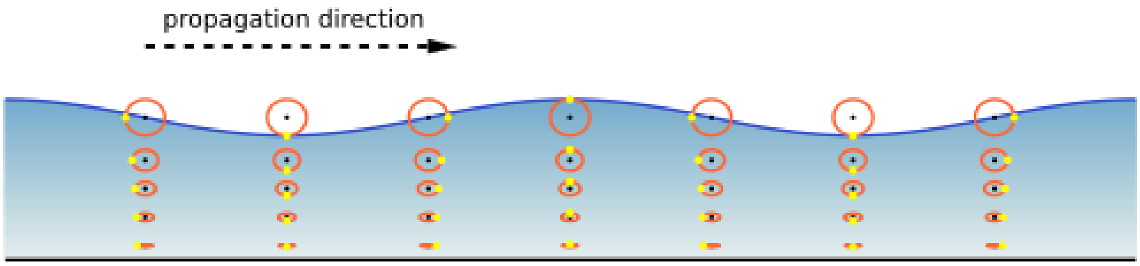

Figure 6 Velocity of water particles under propagating airy waves

Mathematically wave elevation according to Airy wave theory can be expressed as

$$\zeta = a\sin(\omega t - kx)$$

Equation 11

For a wave propagating along the positive *x*- axis.

Waves leads to a time dependent pressure component

$$p_d = \rho g \zeta e^{kz} \sin(\omega t - kx)$$

Equation 12

where ρ is the density of the fluid. For infinite water depth and

$$p_d = \rho g \zeta \frac{\cosh(z+h)}{\cosh(kh)} \sin(\omega t - kx)$$

Equation 13

for finite water depth. *k* is the wave number $k = \omega^2 / g$ for infinite depth and $k\tanh(kh) = \omega^2 / g$ for finite depth. Figure 7 shows pressure under a wave crest and how dynamic pressure and static pressure distributes under a wave crest.

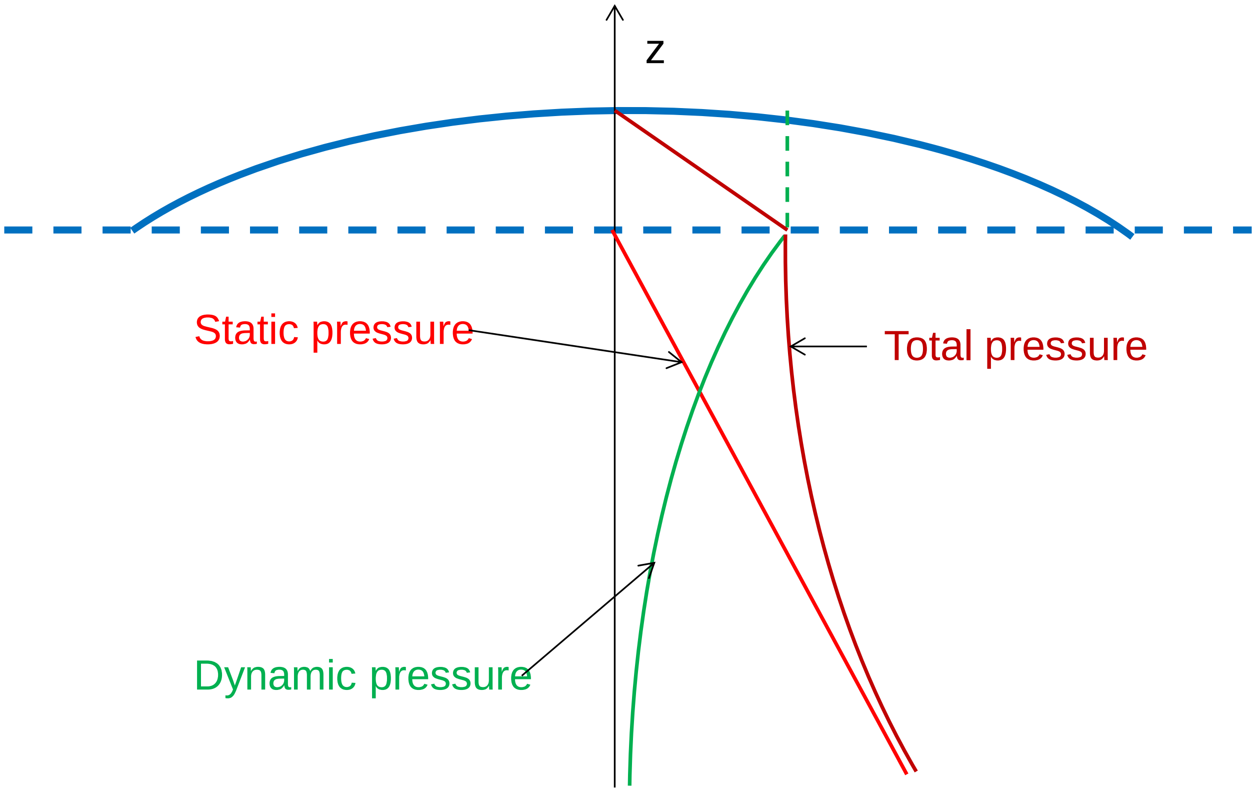

Figure 7 Sea pressure under waves

As seen from Figure 7 the felt total pressure is the static pressure plus dynamic pressure and can be formulated as:

$$p = p_d - \rho g z + p_{atm}$$

Equation 14

Simplified this can be seen as the hydrostatic pressure under the wave crest, but with the effect of the wave decaying with depth. In the area above the mean water line and under the wave crest, the pressure is calculated simply as the hydrostatic pressure under the instant wave crest. This means that the dy. Figure 8 shows the pressure distribution under a wave through.

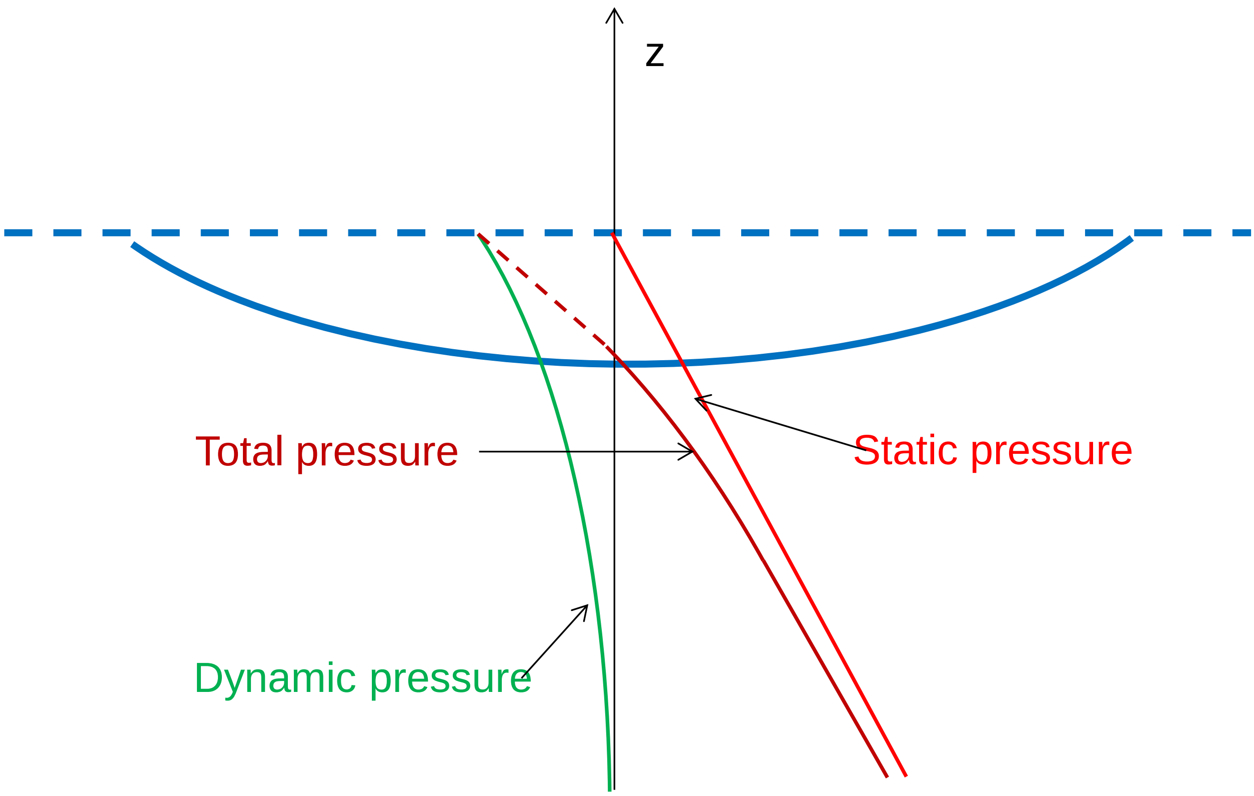

Figure 8 Pressure under wave through

Consider a body submerged in water under waves

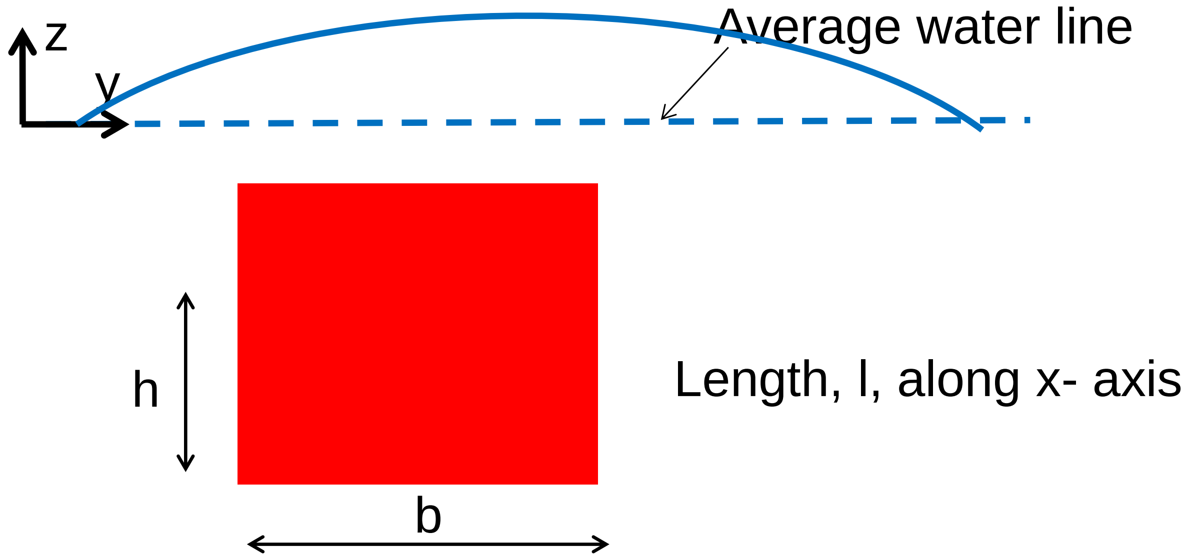

Figure 9 Submerged body

2.2.2. Froude Krylov force

Forces from water to a submerged body will be integral of the pressure around the body. As the static pressure is constant we may integrate the pressure to find the force on the body. We start out with integrating the pressure over the surface, then the Froude Krylov force, F_{FK} is

$$\vec{F}_{FK} = -\iint_{Sw} p\vec{n}ds$$

Equation 15

Where p is the pressure introduced by the undisturbed wave field.

2.2.3. Diffraction force

The pressure under the waves is associated with fluid velocity. This means that in order to keep its position, the body in water will introduce a change in the fluid particle motion on and around the body. For a fixed body, the fluid velocity must be zero normal fluid velocity to the body as shown in Figure 10. The forces caused by the pressure of the undisturbed incident waves are called the Froude Krylov forces. The presence of the body is disturbing the incident waves. The Forces caused by the body´s disturbance of the wave field is called "diffraction forces" and is denoted F_D. The normal velocity to the body for the diffracted wave field is at any time opposite to the velocity caused by the incident wave.

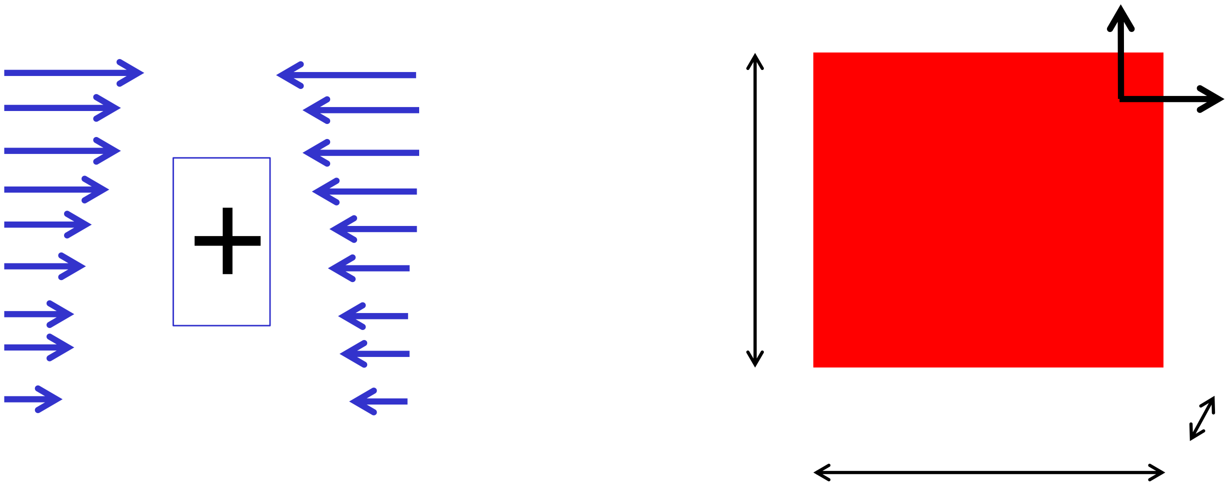

Figure 10 Velocity field around a submerged body

$$\vec{F}_{D} = -\iint_{Sw} p_D\vec{n}ds$$

Equation 16

The total force to the body is then

$$\vec{F} = \vec{F}_{FK} + \vec{F}_D$$

Equation 17

2.2.4. Strip theory for calculation of hydrodynamic forces

In a general case it can be difficult to find the wave field disturbance and then the diffraction force. AquaSim offer one integral possibility to derive this by “strip theory” (Fathi 1996). Strip theory divides the object to strips and solves a numerical problem to find the diffraction force. For an object at the free surface segments are put to both the object and the free surface, and the relation that the normal velocity to each body surface strip must be opposite the velocity of the waves. At the free surface the normal free surface condition applies. This is shown in Figure 11.

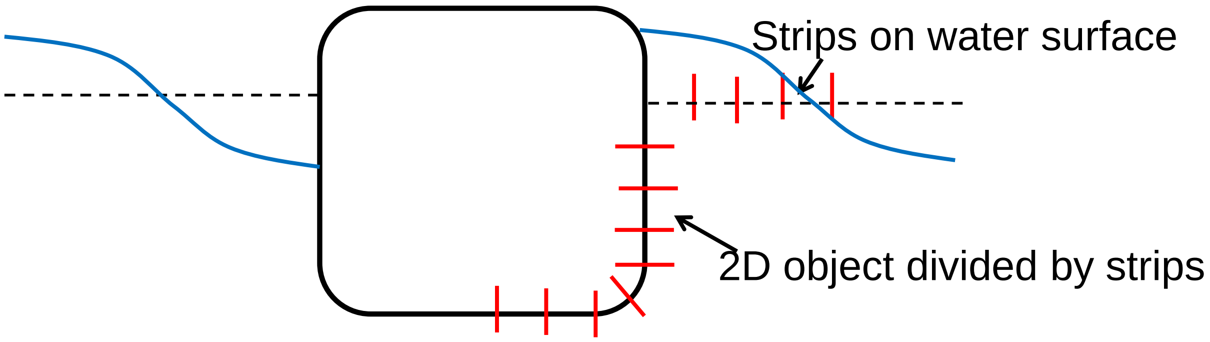

Figure 11 Segments for the “sink source” numerical calculation of diffraction.

Physically the forces to the object are associatted with the fact that part of the incident wave is reflected such that there will be a lower wave on the lee side of the object called the transmitted wave and a wave on the weather side in the opposite direction called the refle

Note that the free surface condition has been linearized to the mean free surface which is consistent with linear wave theory. There are however effects which is not picked up by this approach where the normal most important case for large volume floating objects is wave drift.

2.2.5. Viscous forces

Viscous forces caused by vortices are not accounted for by strip theory of other type of “sink source” theory as it assumes an invicid flow. This can be added to AquaSim by coefficients similar to the coefficients for objects where the Morison load formulation is applied. See Aquastructures (2013) for details.

2.2.6. Cases with long waves relative to structure cross section

With only the dynamic part of the pressure accounted for, Equation 15 can be written as

$$\vec{F}_{FK} = -\iint_{Sw} p\vec{n}ds = \iint_{Sw} \rho g \zeta e^{kz}\sin(\omega t - kx)\vec{n}ds$$

Equation 18

Then by using the Gauss divergence theorem (http://en.wikipedia.org/wiki/Divergence_theorem) and using force in the *x*- direction directions as example on may rewrite Equation 18 to

$$F_{FKx} = -\iint_{Sw} p\vec{n}_x ds = -\iiint_V \frac{dp}{dx}\vec{n}_x dV = -\iiint_V \frac{d}{dx}(\rho g \zeta e^{kz}\sin(\omega t - kx)\rho g \zeta e^{kz}\sin(\omega t - kx))dV$$

Equation 19

And further

$$F_{FKx} = -\iint_{Sw} p\vec{n}_x ds = -\iiint_V \frac{dp}{dx}\vec{n}_x dV = \iiint_V k\rho g \zeta e^{kz}\cos(\omega t - kx)\rho g \zeta e^{kz}\sin(\omega t - kx))dV$$

Equation 20

applying $k = \omega^2 / g$ gives further

$$F_{FKx} = -\iint_{Sw} p\vec{n}_x ds = -\iiint_V \frac{dp}{dx}\vec{n}_x dV = \rho\iiint_V \omega^2 \zeta e^{kz}\cos(\omega t - kx)\rho g \zeta e^{kz}\sin(\omega t - kx))dV$$

Equation 21

which can be expressed as

$$F_{FKx} = -\iint_{Sw} p\vec{n}_x ds = -\iiint_V \frac{dp}{dx}\vec{n}_x dV = \rho\iiint_V a_x dV$$

Equation 22

where a_x is the acceleration of the fluid in the submerged part of the volume. For objects where the wave length is large compared to length of structure, Equation 22 can be simplified as

$$F_{FKx} = \rho V a_x$$

Equation 23

which is the Froude Krylov part of Morison´s equation. a_x is taken at the center of the object. Catenary moored large volume structures is normally not so large that this is the case such that the full integral should be calculated.

2.2.7. Morisons equation

Fluid acceleration is the derivative of fluid velocity. Denote fluid velocity, *u* and its derivative $\dot{u}$ whereas the velocity of the structure is denoted *v* with its derivative $\dot{v}$. Then Morisons equation reads:

$$F = \rho V\dot{u} + \rho C_a V\dot{u} - \rho C_a V\dot{v} + \frac{1}{2}\rho C_d A(u - v)|u - v|$$

Equation 24

Where C_a is the added mass coefficient and C_d is the drag coefficients which are parameters set empirically or analytically. Description can be seen at http://en.wikipedia.org/wiki/Morison%27s_equation The terms in Equation 24 are:

$\rho V\dot{u}$ is the Froude Kriloff force. This term is added not only in the *z*- direction, for also in the horizontal plane.

$\rho C_a V\dot{u}$ is the diffraction force.

$\rho C_a V\dot{v}$ is the added mass.

$\frac{1}{2}\rho C_d A(u-v)|u-v|$ is the drag force.

V is the submerged volume and *A* is the area fronting the fluid motion.

2.3. Added mass and damping

In the previous section forces applying to a (partly) submerged body being still was introduced. This section described the case where the body moves. For a moving body in water, the boundary value problem must solved where the velocity normal to the vessel is not 0 but the velocity of the object.

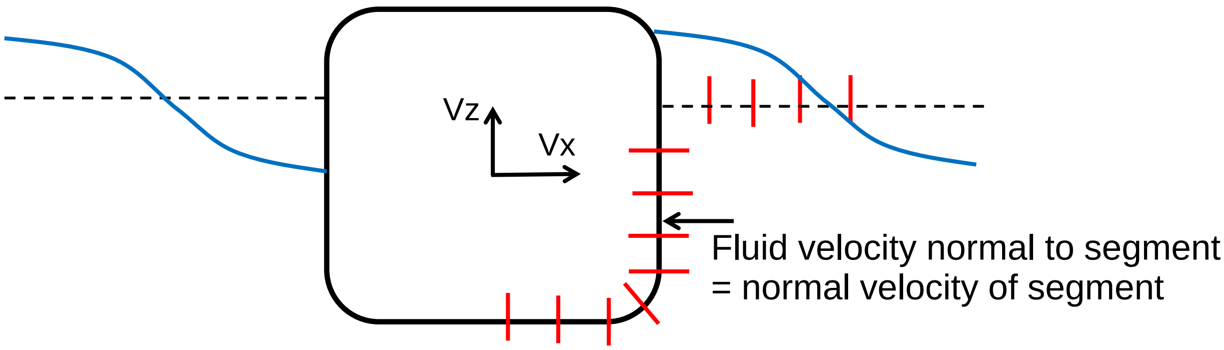

Figure 12

From this boundary value problem, forces are found which are proportional with acceleration and velocity of the object. Forces proportional with acceleration is called “Added mass” and proportional with velocity is called “damping”. Hydrodynamic damping is associated with waves moving outwards from the object called radiating waves.

The solution for added mass and damping will depend on the wave period.

2.4. Wave drift forces

Wave drift forces are constant (in regular waves) or slowly varying forces (irregular seas) proportional to the square of the waves reflected by a vessel. This follows Maruos formulae (See e.g. Faltinsen 1990). Consider a 2 dimensional case. The average drift force over a time period is assumed being according to Maruos formulae:

$$\overline{F2} = \frac{\rho g}{2} A_r^2$$

Equation 25

Where $\overline{F2}$ is the second order force averaged over a wave period. ρ is the density of water. *g* is the gravity constant. A_r is the amplitude of the reflected wave. Using this information, *F2* can be calculated as:

$$F2 = \rho g A_r(t)^2$$

Equation 26

It is seen that Equation 26 will give the same average force as Equation 25. *F2* will always act normal to the hull in in the opposite direction of the reflected waves. The reflected waves are the reflected wave calculated by the strip theory or other numerical method accounting for waves originating both from diffraction and radiation. In irregular waves, the reflected wave is calculated for each period in the spectrum. In this calculation the reflected wave is a combined wave from diffraction and radiation.

Equation 26 has an advantage over Equation 25 since it will work fine for irregular seas as it uses the instantaneous wave elevation at the ship side. Hence the slow drift time variation of this load term is automatically accounted for. Equation 26 has also a load component in sum frequency. This component is correct in magnitude but not formally correct in phase.

A more detailed description of this is given in Aquastructures (2012).

2.5.End effects, 3D effects

Strip theory accounts for variation over the hull in the x- direction. However this variation should be small compared to the distance along the x- line. For a ship shapes structure as shown in Figure 31 this is a very good assumption. However for barges with a square shape, strip theory not account for the ends which in such cases is a significant part of the hull.

Normally the ends are modeled with transverse elements applying the Morison equation to them even though they do not fulfill the relation that wave length should be large compared to object. As long as the waves move transverse to the vessel (in the y- direction) this is fine. However for when the waves starts to vary with the *x*- direction this is no longer valid. Hence there is a possibility for using strip theory in two directions simultaneously in AquaSim. The forces in one of the directions are calculated by "2D" strip theory which means only horizontal forces are accounted for. This means it can be used in combination with the usual strip theory without doubling up vertical forces.

Using 2D strip theory in AquaSim, only forces in plane are accounted for whereas vertical forces and added mass is disregarded. This may be applied to the ends and combined with "normal" srip theory in the other direction then a quasi "3D" solution is established. The wave loads will be a bit higher than with full 3D since the corner is not accounted for. That is to the conservative side.

3. CASE STUDIES TO IMPORTANT LOAD RESPONSE EFFECTS

This section describes some important response modes for barges and how analysis may be carried out to find if moorings and structure can cope with the load-response.

3.1. Finding natural periods roll and heave

Analysing a barge or any other vessel it is of interest to know the natural periods of the barge.

3.2. High loads in moorings due to impact loads from time varying stiffness of moorings

A typically occurring load condition for moored system is that the system moves and moorings change stiffness. This may lead to large accelerations and forces in the system including the moorings. Due to the ability to handle large nonlinear effects, AquaSim may be used to investigate the occurrence and magnitude of such load-response. This effect occurs typically for any kind of moored structure but is mostly predominant for moored structures with high mass such as barges, ship shaped structures and offshore platforms. It also typically occurs to buoys and similar surface penetrating floats.

3.2.1. Simplified case for validation and comparison to analytic formulae

In order to demonstrate the effect and validate the AquaSim analysis capabilities, a case study with a float connected to a fixed point by a rope as shown in Figure 13 has been established. Consider the particular time instant when the distance rope goes from slack (seen in Figure 14) to straight as seen in Figure 13.

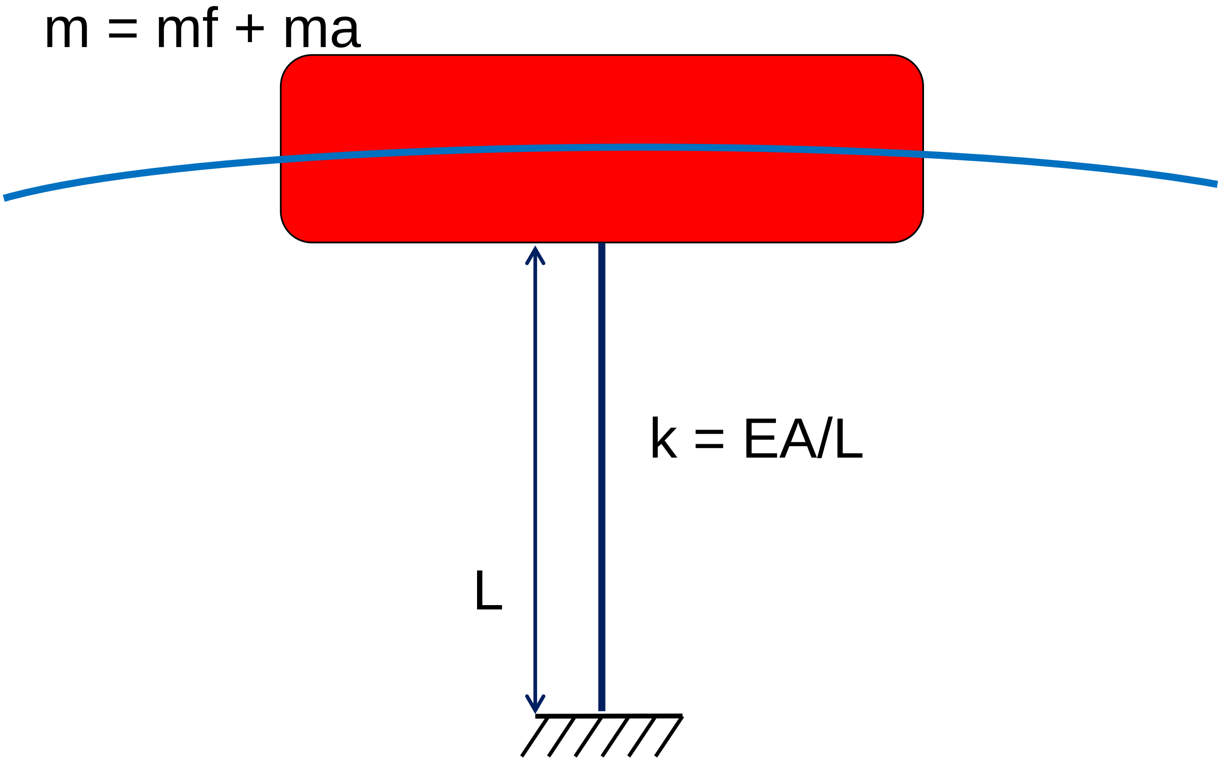

Figure 13 Float attached to bottom with rope

In Figure 13, k is the (spring) stiffness of the rope, E is the Young modulus, A is the (nominal) cross sectional area of the rope and L is the rope length. m = mf + ma is the mass of the float where mf is the mass of the float itself and ma is the added mass which is the hydrodynamic load proportional to the float acceleration. Assume that the mass and added mass of the rope itself is much lower than for the float and hence can be neglected.

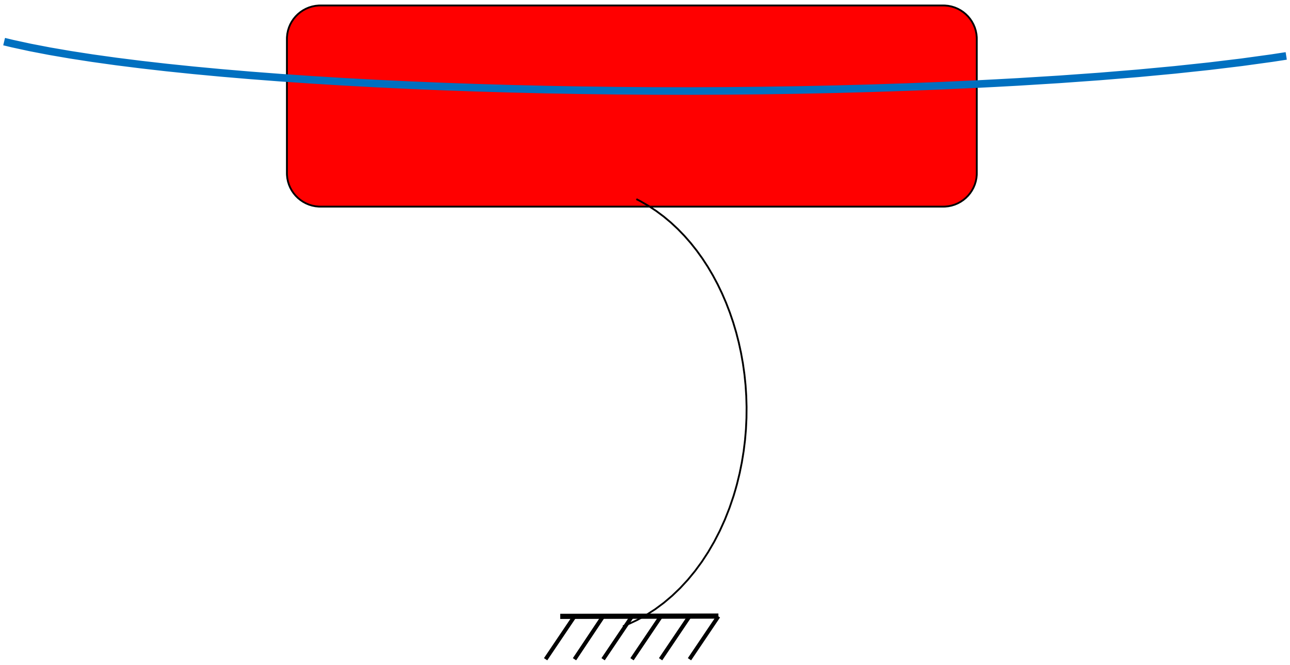

Figure 14 Float with slack rope

Consider a time instant when the float rope has become slack as seen in Figure 14. Assume that the float moves upwards with the wave elevation. At some point, the distance rope goes from slack to straight as seen in Figure 13. As a simplification one can assume that the vertical stiffness of the rope is 0 when the rope is slack and that it is EA/L when it gets stiff and the condition in Figure 13 applies. At that moment, an impact load is introduced to the float.

Assume that the response from this impact load can be described with the classic impulse response function:

$$m\ddot{z} + kz = 0$$

Equation 27

where *z* is the vertical displacement. As a simplification a one dimensional system where the motion in this case is assumed to be vertical in direction parallel with the distance rope is considered. Define the coordinate system such that vertical motion *z = 0* when *t = 0* as shown in Figure 15.

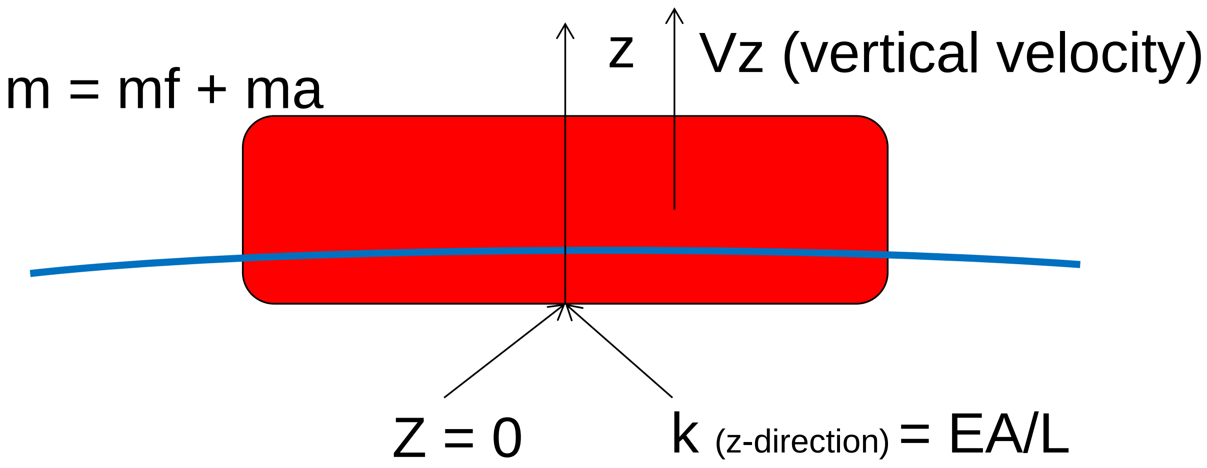

Figure 15 Initial value condition at moment rope goes stiff

Neglect damping in the system (as has been done in Equation 27). This means the system can be solved by applying the classic impulse response equation solution for the motion:

$$z(t) = a\sin(\omega t)$$

Equation 28

where $\omega^2 = k/m$ and a is the amplitude of the response. k is the stiffness of the system caused by the rope holding the float back like a spring. $k = EA/L + k_{aw}$ where k_{aw} is the water plane stiffness $= \rho g a_w$ where a_w is the water plane area. This is a linear solution where the stiffness and mass is assumed to be time invariant. The velocity is the time derivative of the displacement:

$$\dot{z}(t) = a\omega\cos(\omega t)$$

Equation 29

where $\dot{z}(t)$ is the vertical velocity of the system. In our case we have an initial velocity v_0 which is the velocity at the initial time ($t = 0$ (exactly when the ropes gets stiff). Assume that the velocity of the float follows the vertical velocity of the wave elevation as the wave is built up. Then v_0 can be found from the velocity of the vertical wave elevation at the moment the rope snap. Assume a regular wave with amplitude η. The wave elevation can be expressed as

$$\xi = \eta\sin(\omega_e t)$$

Equation 30

where ω_e is the wave frequency of encounter, this frequency has nothing to do with the eigenfrequency of the rope and float. Applying Equation 28 to time step $t= 0$ we get

$$v_0 = \eta\omega e\cos(\omega e t_{t=0})$$

Equation 31

Now assume that the float rope snaps when v_0 is at its max possible value, $v_0 = \eta\omega_e$: In the present model the geometry is such that the rope will be stiff at approximately $\xi = 0$ which is the time the vertical velocity v_0 is at its maximum.

$$v(0) = \eta\omega e = \dot{z}(0) = a\omega\cos(\omega 0) = a\omega$$

Equation 32

This means the amplitude, a is found as

$$a = \frac{v(0)}{\omega}$$

Equation 33

Now the relation between eigenperiod and mass and stiffness is introduced ($\omega^2 = k/m$) to Equation 33:

$$a = \frac{v(0)}{\omega} = \frac{v(0)}{(k/m)^{1/2}} = v(0)(m/k)^{(1/2)} = v(0)\sqrt{m/k}$$

Equation 34

This means that an impact as described above will introduce a harmonic impact response with amplitude as given in Equation 34.

From the maximum response amplitude, the maximum force can be derived as

$$F_{\max} = ka = kv(0)\sqrt{m/k} = v(0)\sqrt{mk}$$

Equation 35

This means the maximum force is proportional to the initial velocity and the square root of the mass and stiffness. From Equation 31 it is seen that the initial velocity is proportional to the wave amplitude η and the wave frequency of encounter, ωe. Introducing $v_0 = \eta\omega_e$ to Equation 35 F_{max} can be expressed as

$$F_{\max} = \eta\omega e\sqrt{mk}$$

Equation 36

Assuming the stiffness of the rope is so much larger than the water plane stiffness, the following equation can be derived:

$$F_{\max} = \eta\omega e\sqrt{mEA/L}$$

Equation 37

Consider a case with parameters shown in Table 3.

Table 3 Main data for system and analysis

Float data	
Float length [m]	5.4
Float volume [l]	5132
Float circular diameter [m]	1.1
Float weight [kg]	2907
Rope data	
Length [m]	10
Cross sectional area [mm2]	1000
E-modulus [Mpa]	10000
Environment data	
Wave period [s]	6
Current velocity n[m/s]	1
Added mass [kg]	4737

A model has been established in AquaSim for the case presented in Table 3. The model is shown in Figure 16.

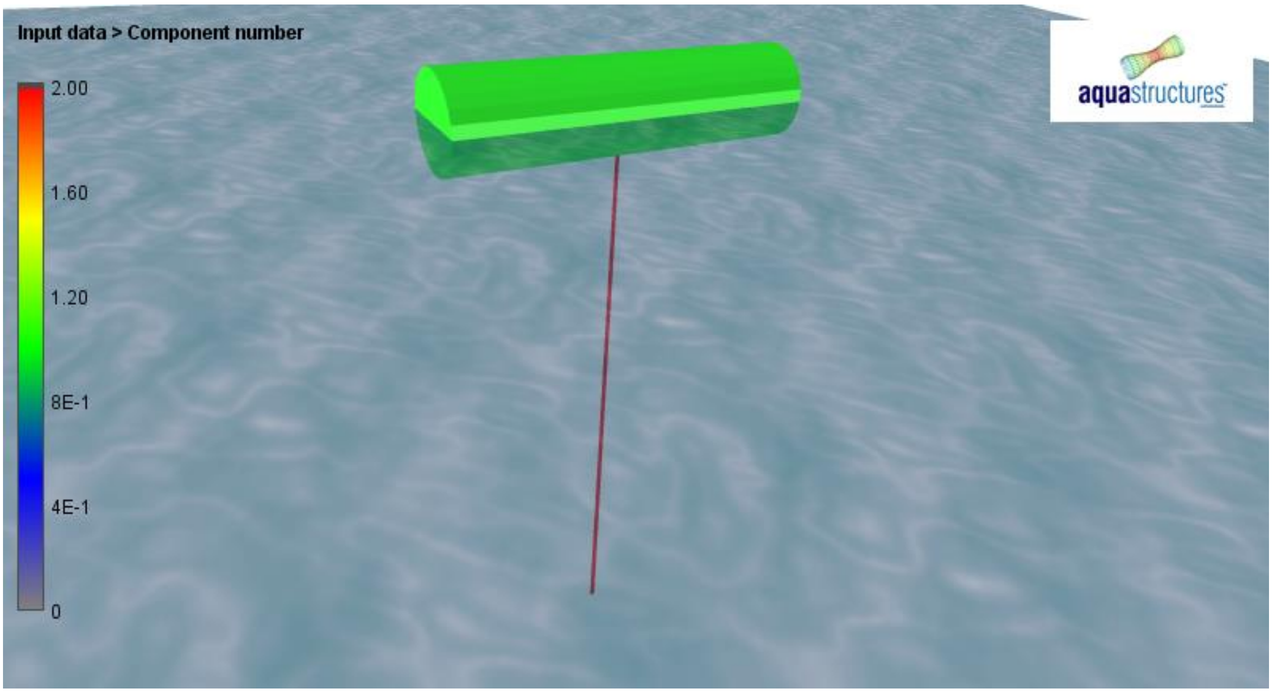

Figure 16 Analysis model, impact load test case

In the analysis waves and current is from the left to the right along the positive x- direction as shown in Figure 17.

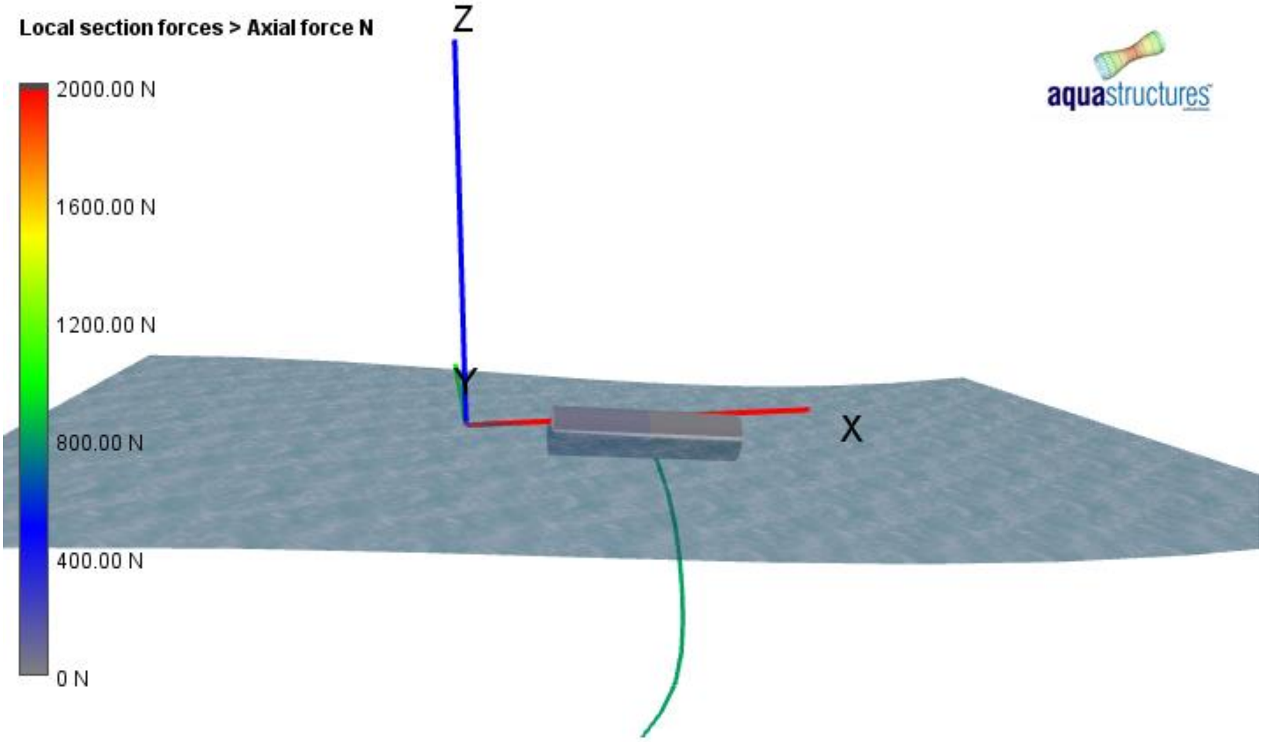

Figure 17 Analysis model, wave and current direction is along positive x- axis.

Analysis has been carried out with varying wave amplitudes and compared to Equation 36. This is shown in Figure 18. In this figure, the labels mean:

- Analytic formulae: Max load calculated from input from Equation 36.
- Peak load AquaSim 1: Max load calculated with AquaSim with the analysis model taking in and out of water into account.
- Peak load AquaSim 2: Max load calculated with AquaSim with the analysis model not taking in and out of water into account.
- Analytic formulae EA/L: Max load calculated from input from Equation 37 neglecting the water plane stiffness. For this case the water plane stiffness is 5.6% of the total stiffness.

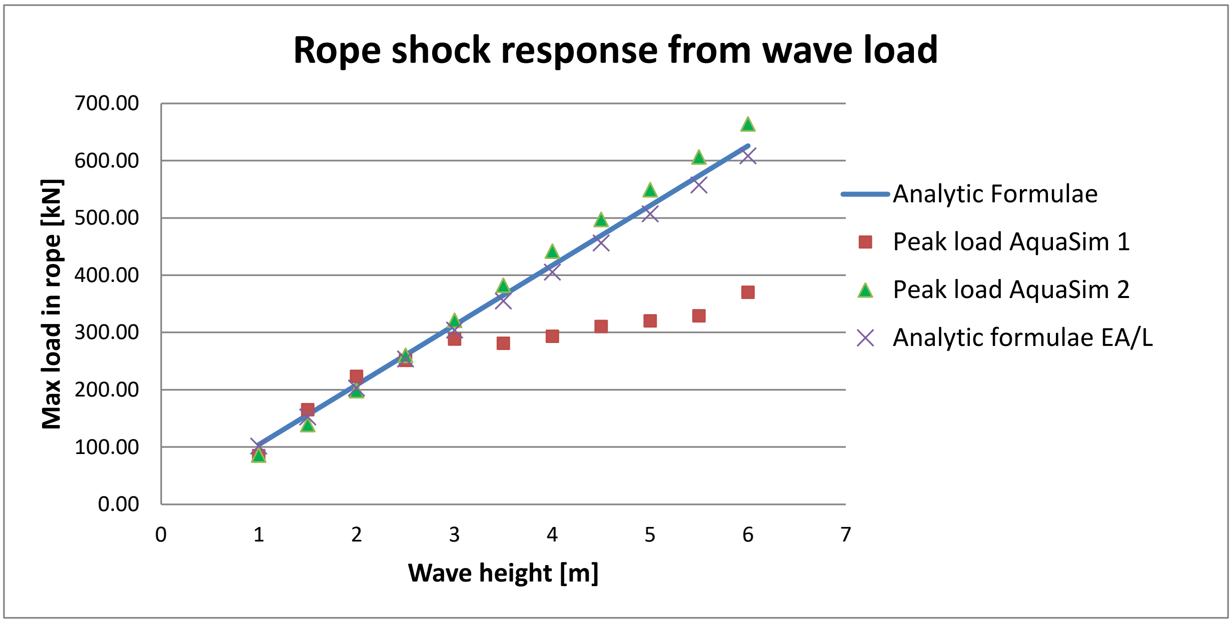

Figure 18 Maximum axial load in rope

As seen from Figure 18, the results by Equation 36 and the AquaSim 2 model shows very good correspondence for all wave heights. Figure 19, Figure 20 and Figure 21 shows how the impact load strikes the rope as the float is moved from slack rope to the rope getting stiff.

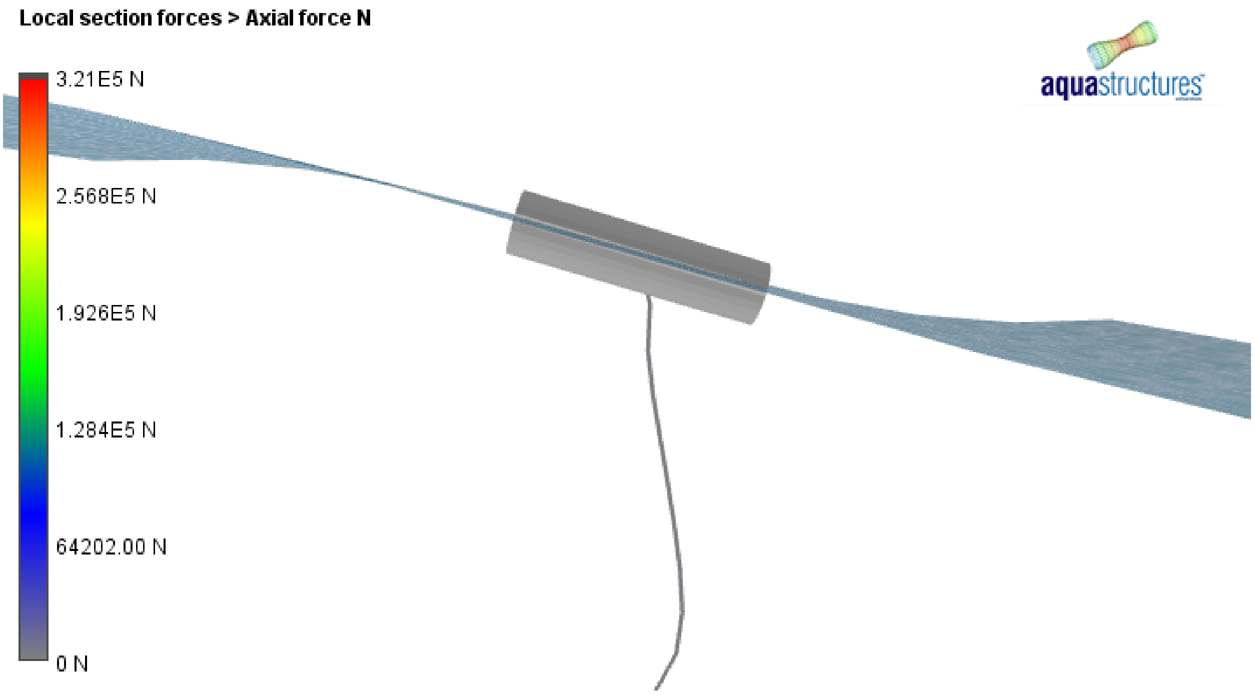

Figure 19 Float when rope is slack. There are no forces in the rope

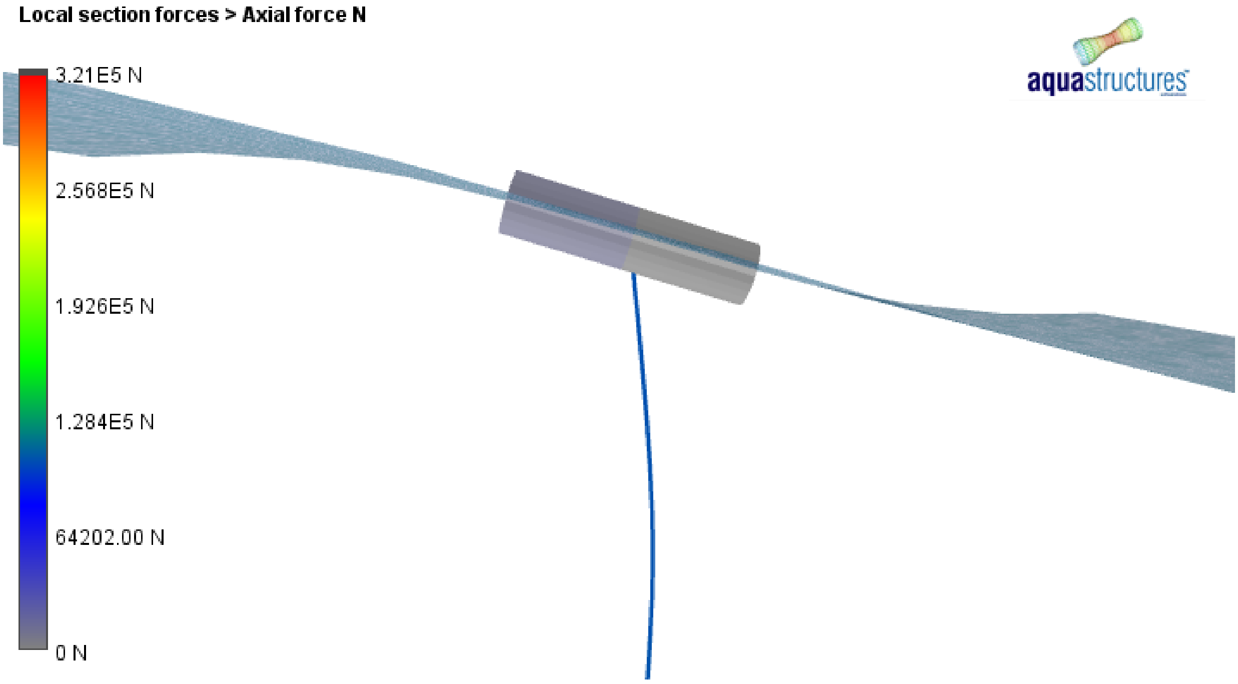

Figure 20 Forces commencing in rope as float is moved upwards by wave motion

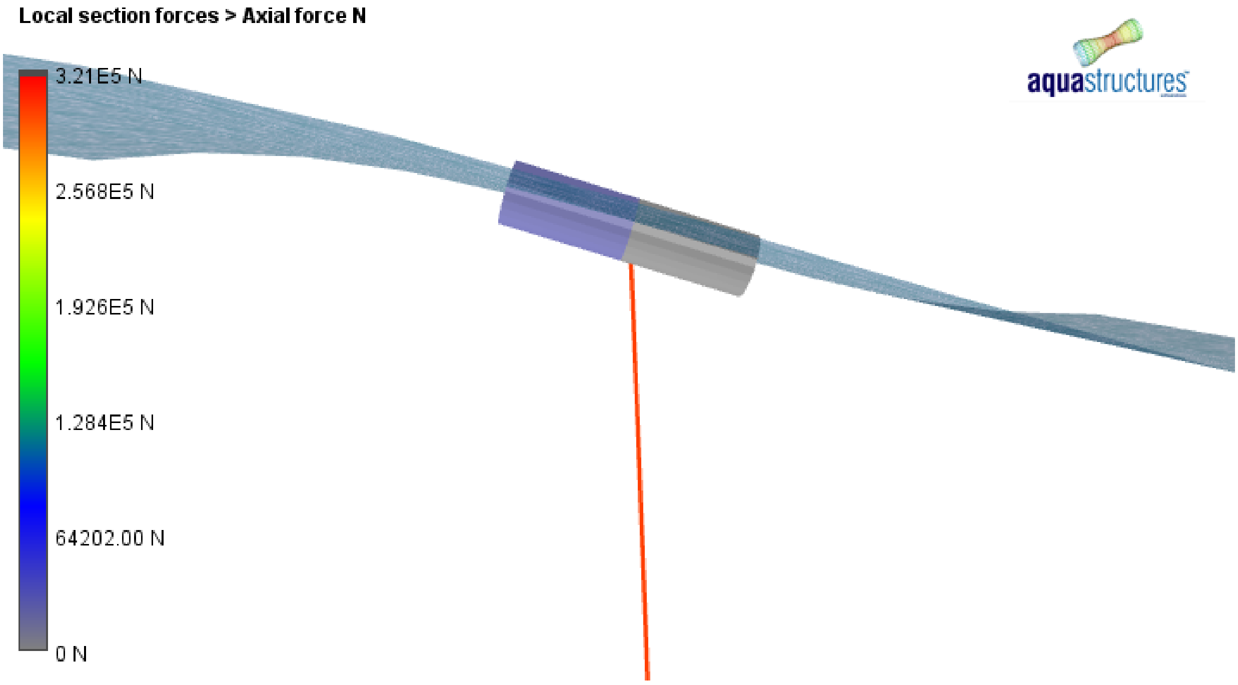

Figure 21 Impact load to rope as the float has been moved so much upwards that the rope gets stiff

The time series response for the axial force in the rope is in the AquaSim analysis is shown in Figure 22 and Figure 23.

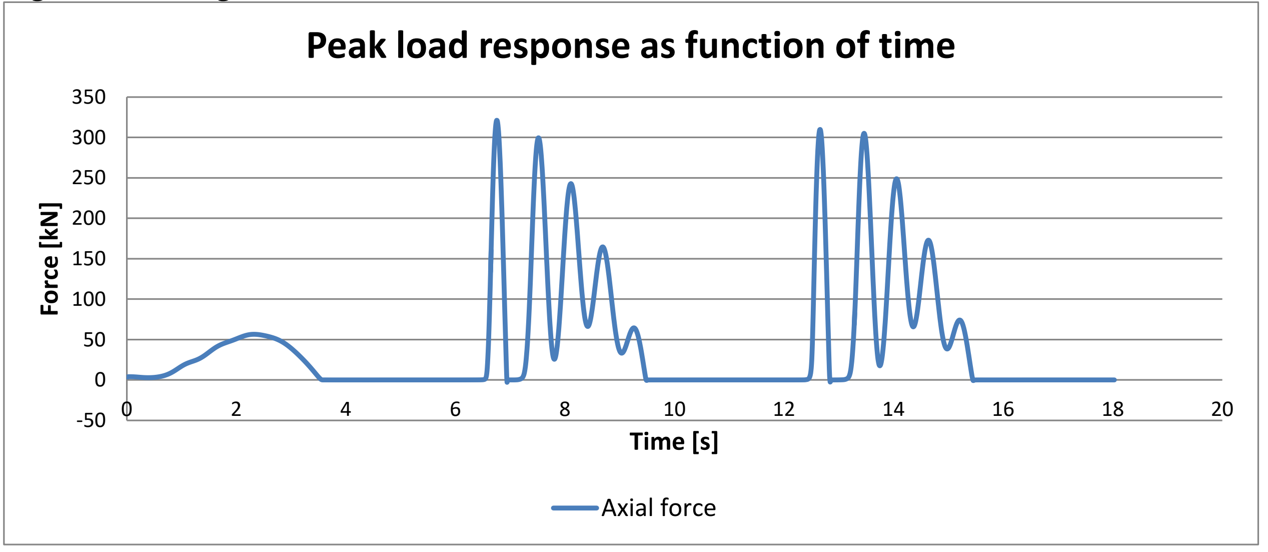

Figure 22 Time series for the AquaSim analysis of the axial force

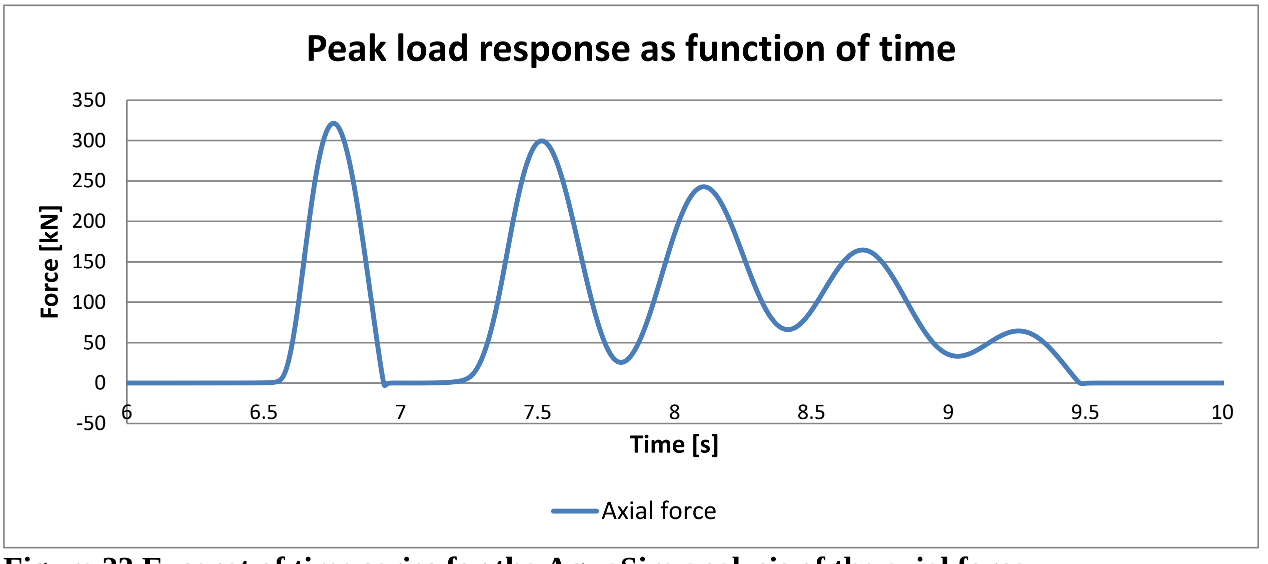

Figure 23 Excerpt of time series for the AquaSim analysis of the axial force

As seen from Figure 22 and Figure 23 the load is at 0 and then increase sharply due to the rope going from slack to stiff. As seen from the figures, a harmonic response is then decaying over time. As seen from Table 4 the natural period of a swinging system with mass and added mass of the float and stiffness of the rope is approximately 0.53 s.

Table 4 Key data for the natural period of the rope

Name	Abbreviation	Value
Length	L	10
Youngs module	E	1E+10
Cross sectional area	A	0.001
Stiffness EA/L	k	1000000

Water plane stiffness	kw	59728
Total stiffness	ktot	1059728
Total mass	m	7644
omega		11.77
Period	s	0.53

The natural period of 0.53 seconds corresponds very well to the response seen in Figure 23 apart from the first succeeding cycles where the time between succeeding peaks are longer. That is plausible as when inspecting Figure 23 the load gets to 0 between the first and the second response cycle after the impact. This is because the response amplitude is so large the rope gets slack. In that case the stiffness decreases and the natural period increase. With only water plane stiffness, the natural period is 2.25 sec.

This analysis case shows that AquaSim manage to calculate the peak loads occurring in mooring lines as they goes from slack to stiff. This is an important design criteria for a wide range of moored structures and equipment.

3.2.2. Barge example

Inspecting Equation 35 is seen that the peak force in the mooring line is proportional to the velocity of the floating mass at the time the mooring line goes from slack to stiff. It is also proportional to the square root of the mass and the stiffness.

Consider moored barge as shown in Figure 1. The mooring system of this barge is a typical mooring system with ropes with chains at each ends. To the bottom chains are normally used to avoid chafing when varying environmental conditions leads to varying parts of the mooring line that lies on the seabed. The basic idea is to avoid rope connection to the sea bed. Close to the vessel chains are normally also used. Using weights on a mooring line also means changing the moorings stiffness properties. It can be used to make the mooring go more softly from a slack condition to a stiff condition. How the system responds as one mooring goes from slack to stiff is shown in Figure 24 - Figure 27.

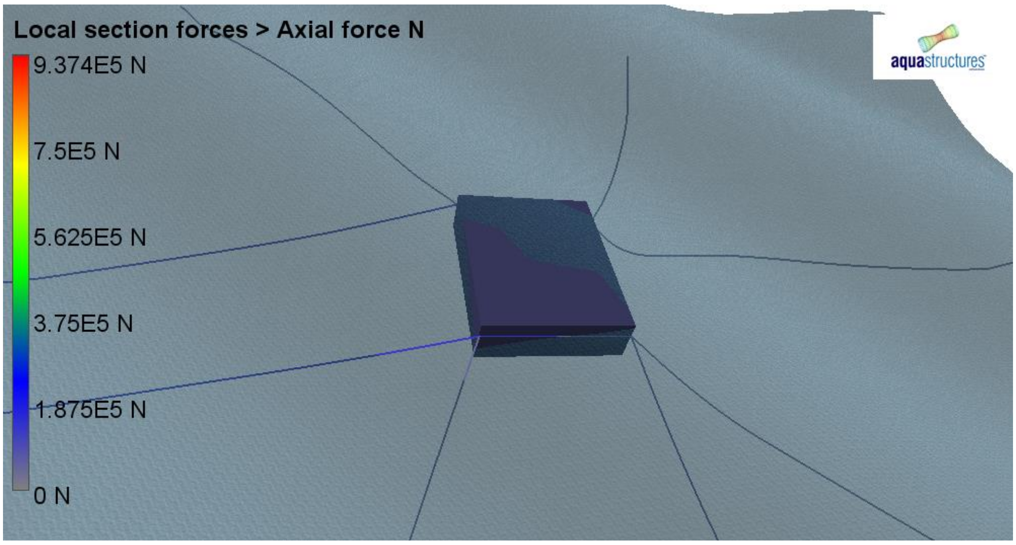

Figure 24 Waves loads acting on barge. Loads in mooring lines are indicated by colour

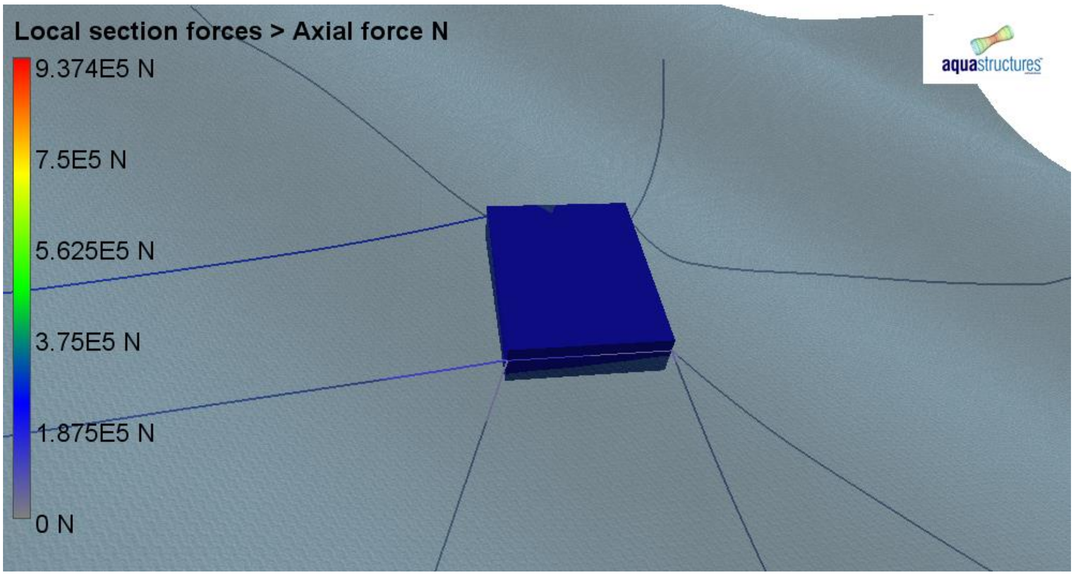

Figure 25 Mooring line loads 0.64 seconds later than Figure 24

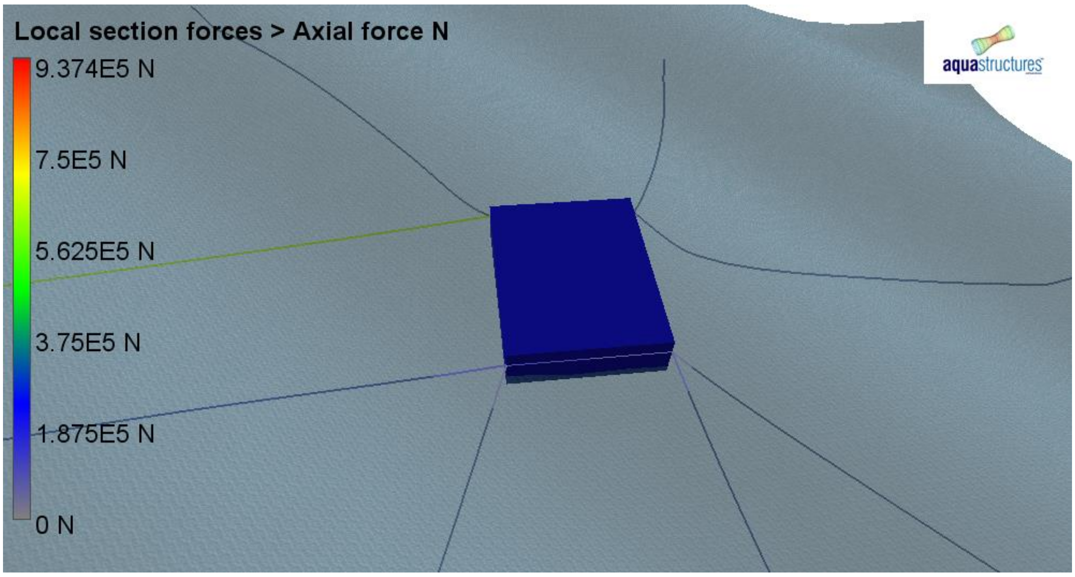

Figure 26 Mooring line loads 0.64 seconds later than Figure 25

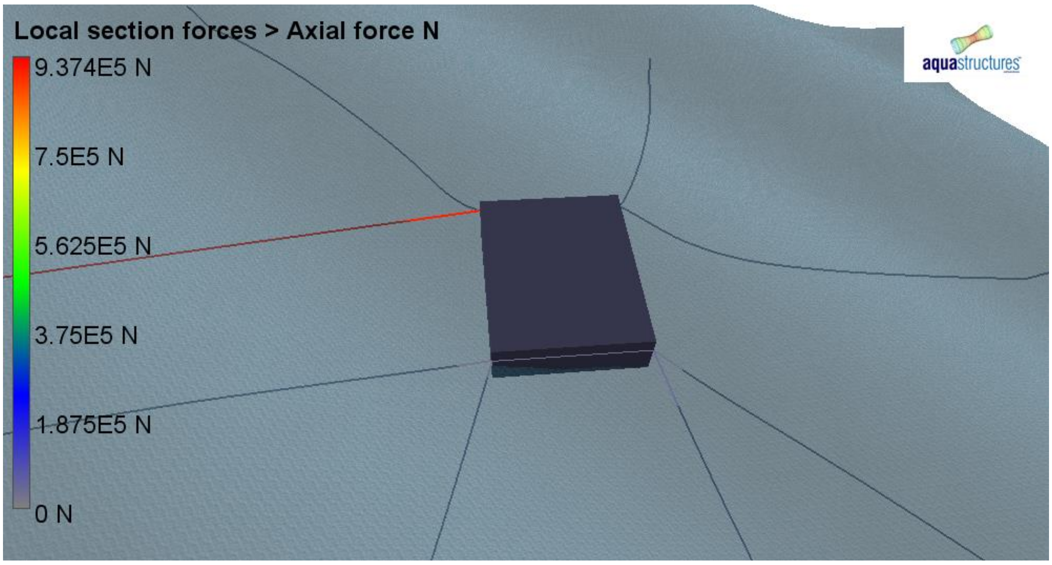

Figure 27 Mooring line loads 0.64 seconds later than Figure 26

Figure 24 - Figure 27 shows how one of the mooring lines gets a peak load when the mooring line goes from slack to stiff. This happens in oblique seas.

The load effect seen in Figure 24 - Figure 27 is normally the most important load effect for the design of a mooring system for barges such as seen in these figures. With respect to loading it is normally the highest wave that causes this response. Note that this is valid for steep waves. For swell, the load will normally be smaller as the barge velocity is smaller.

Although the highest shock load response happens for the largest wave it is important to include also other load contributors such as currents, wave drift and wind. It is normally accurate enough to include these other effects by a constant load. Wave drift may introduce a separate quasi resonant motion. This is considered in a separate section of this report.

The shock load effect considered in this section is always accounted for by running a time domain analysis in AquaSim. It is however of large importance to run high enough density on time steps to cover the peak load. It is also harder to find convergence at such load peaks due to the nonlinear characteristics of the load response.

This response mode can sometimes be indicated in AquaSim by seeing non-convergence. Then more analysis should be run with fines time steps to find the finer characteristics of the response.

3.3. Wave drift forces and response

In AquaSim it can be chosen to account for wave drift forces.

3.3.1. Case study in Aquastructures (2012) for validation

Figure 28 shows a case study analysed with drift and reported in Aquastructures (2012).

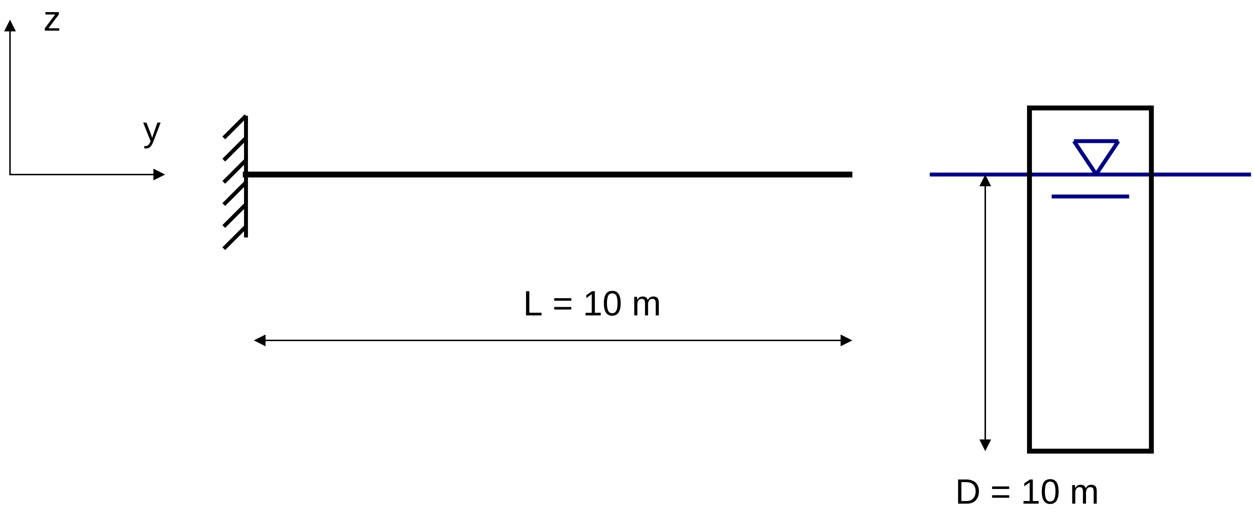

Figure 28 Data used for case study where data are compared to data using the reflected wave asymptotic results. The depth below the water of the beam is 10 meters and the width of the beam is 1 meter.

As seen from Figure 28 the structure is mounted like a cantilever beam. The resulting horizontal force in the *y*- direction is shown in Figure 29

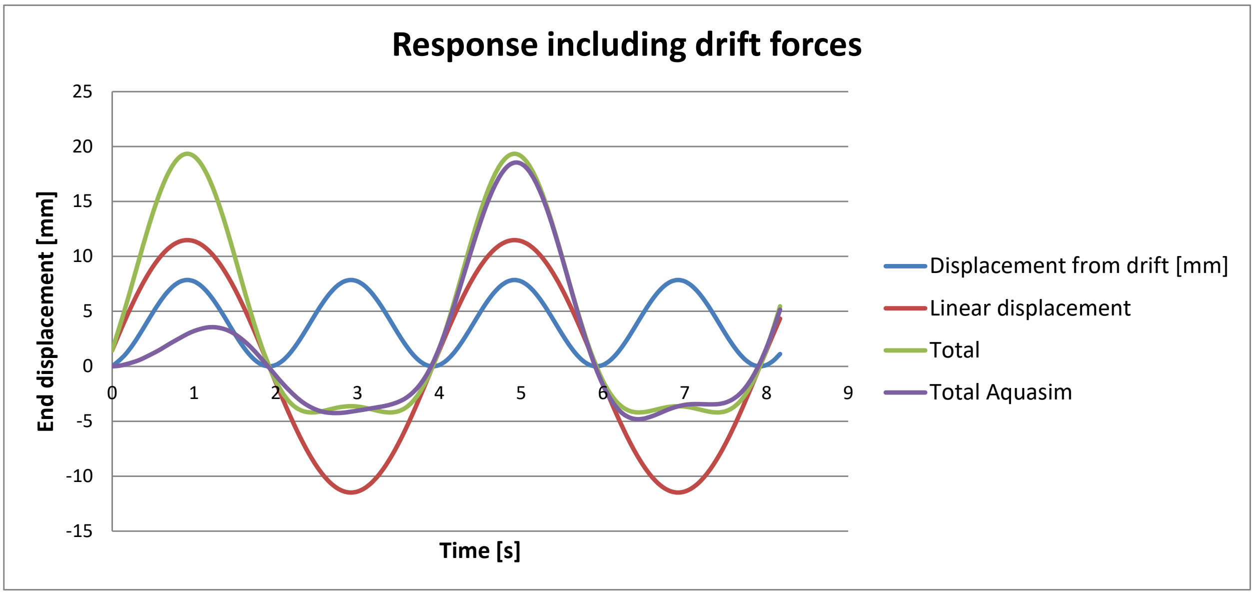

Figure 29 Response from wave with wave amplitude 5 meter and wave period 4 seconds.

From Figure 29 it can be seen that the drift force calculation in AquaSim compares well with analytical calculation. It is also seen how the drift force is applied in time by calculating the square of the reflected wave at each time-step. This means the time dependence of the wave drift force is accounted for in irregular sea. As seen from Figure 29 there is also a sum-frequency component in the drift force calculation. In order to have this correct a more refined approach should be established for this load component. As it is now, it is however very fine to detect possible resonance effects on sum frequencies.

3.3.2. Slow horizontal motion in irregular seas

In irregular seas, the wave drift force varies with the envelope (http://en.wikipedia.org/wiki/Envelope_%28waves%29) of the reflected wave where both the upper and lower envelope in a general case is seen in Figure 30.

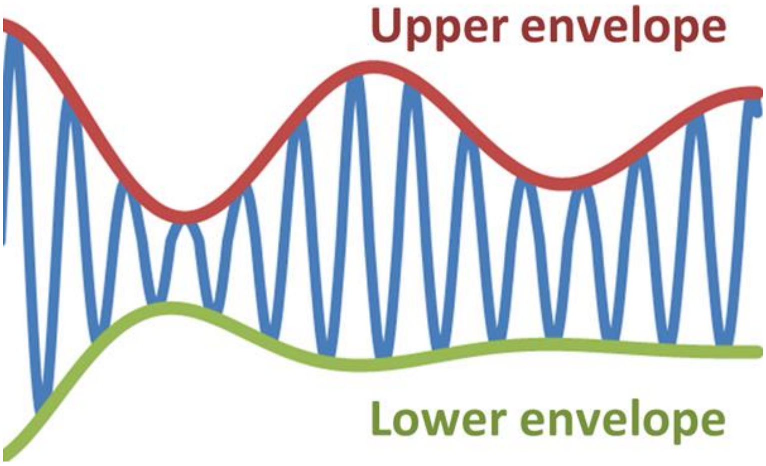

Figure 30 Upper and lower envelope of a time series.

This envelope effect could be of importance if the mass of the floating object and the stiffness of the mooring system is such that the natural period of the system coincides with the natural period of the envelope.

The stiffness of the mooring system is in general nonlinear. Hence there is no single value natural period. In order to find horizontal stiffness of mooring system one should apply static loads to derive displacement to figure out the stiffness as the derivative of the change in load to the change in displacement.

3.3.3. Combination wave drift and current

The current velocity is accounted for by formulae 5.22 in Faltinsen (1970) as

$$F_2 = F_2(1+\frac{\omega U Cos\beta}{g})$$

Equation 38

F2 is the drift force as outlined in Equation 25. As seen from Equation 38 current parallel to the direction of the waves increase the drift force. Equation 38 is implemented to AquaSim.

3.3.4. Direction of drift forces to sections

The formulation used in strip theory assumes implicit that the angle Φ seen in Figure 31 is small. When calculating drift forces the angle Φ if found and accounted for when the drift force and the direction of the drift force is derived by

In case waves are not perpendicular to the “upstream” line where the vessel side intercepts the water, the drift force is corrected by

$$Fac_c = \sin^2(\beta)$$

Equation 39

where β is the angle between the incident wave and the vessel water intersection line. β, is the angle between the normal of the vessel side and the wave direction as shown in Figure 31.

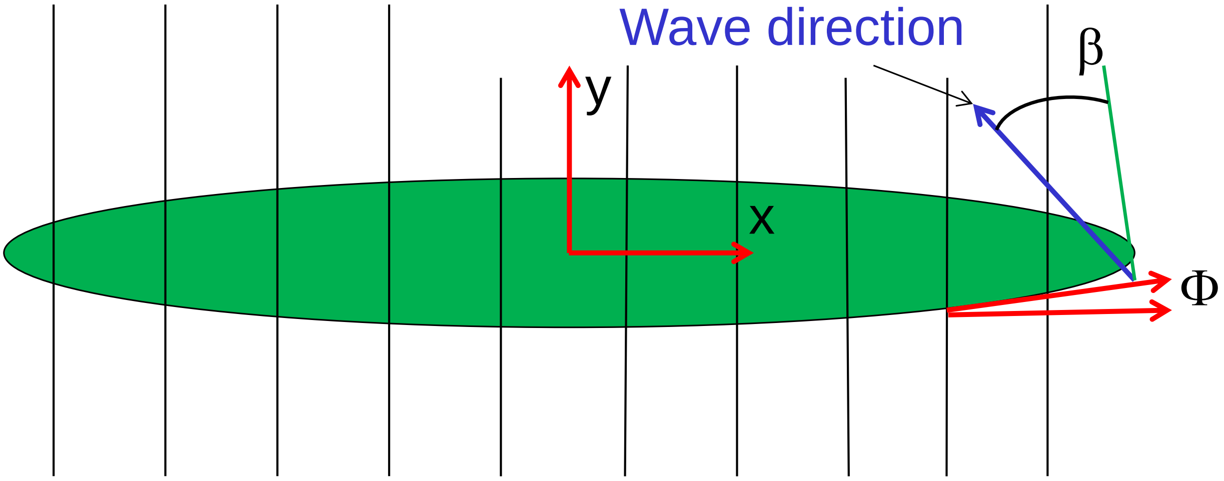

Figure 31

3.3.5. Analysis case applying drift loads in AquaSim

An Aquasim analysis model of a barge as shown in Figure 32 has been established

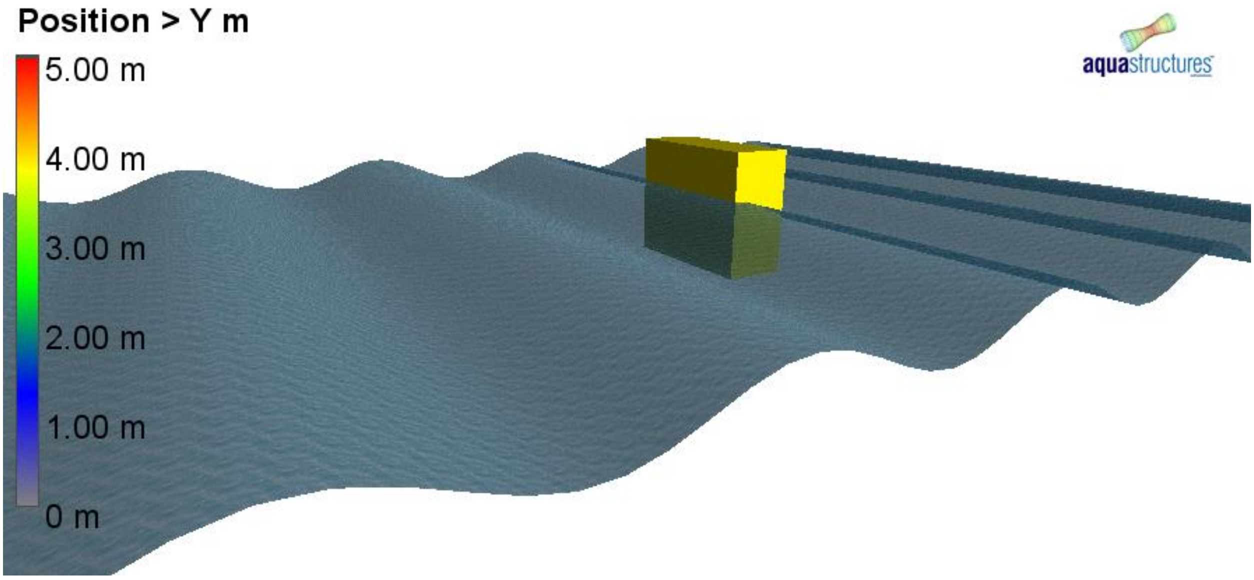

Figure 32 barge seen in regular waves

This system is seen in Figure 33.

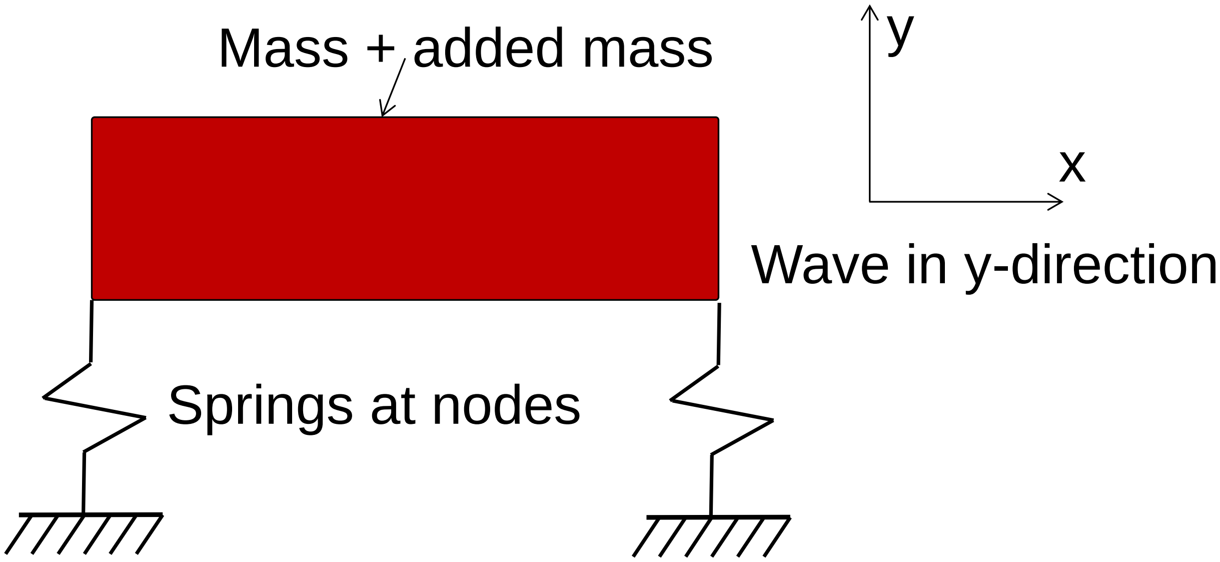

Figure 33 Barge with springs at ends

The particulars of the system are given in Table 5.

Table 5 Main particulars for barge

Parameter	Value
Length [m]	10.0
Width [m]	4
Depth [m]	4
Deplacement [m2]	160
Weight [kg]	164000
Spring stiffness total [N/m]	5000

More relevant data is given in Table 6.

Table 6 Data for barge

System data parameter	Value
Mass [kg]	164000
Horizontal added mass at analysed wave period [kg]	23660
Mass total [kg]	187660
Spring stiffness [N/m]	5000
Eigenperiod [s]	38.49
Wave period [s]	3.00

As see from Table 6 the wave period is short compared to the moulded depth of the vessel. This means on can assume that most of the incident wave is reflected.

Table 7 Analytic calculation of displacement from drift force

Parameter	Value
Amplitude incident wave [m]	1.00
Amplitude reflected wave	0.98
F2 per meter [N/m]	4848.26
F2 total [N]	48482.59
Displacement by F2 [m]	9.70

Figure 34 shows response calculated by AquaSim for a regular wave with period 3 sec. As seen the average drift is between 9.5 and 10 meters which is good correspondence with the analytic average value in Table 7.

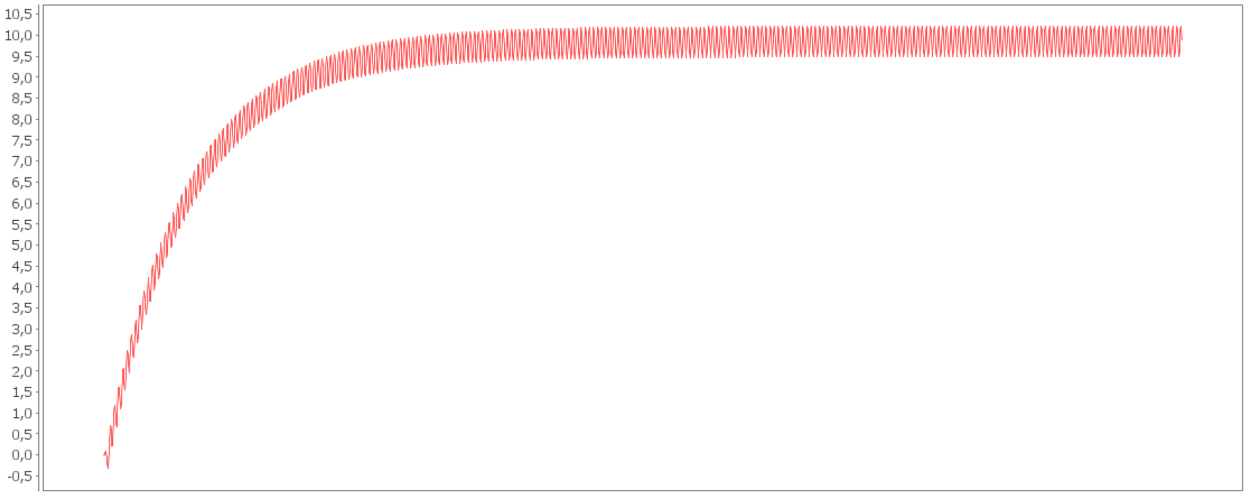

Figure 34 Response in regular waves. Displacement *y*- [m]. Length of time series is 13.125 minutes.

Figure 35 shows a time instant in analysis of an irregular sea and Figure 36 shows the time series of the displacement.

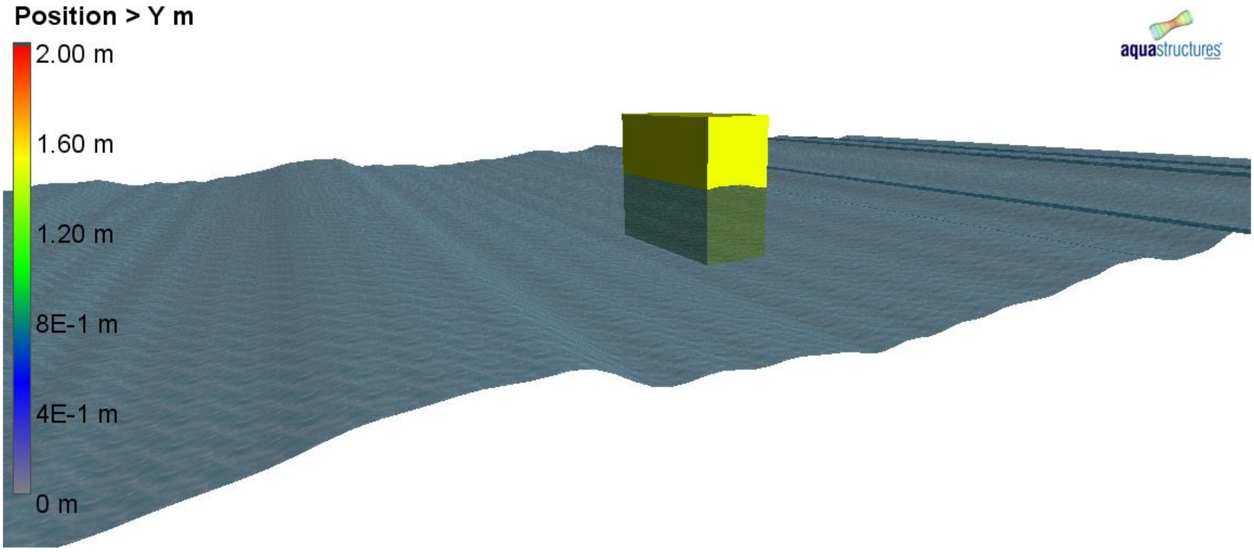

Figure 35 Irregular long crested waves. Jonswap spectrum γ = 3.3.

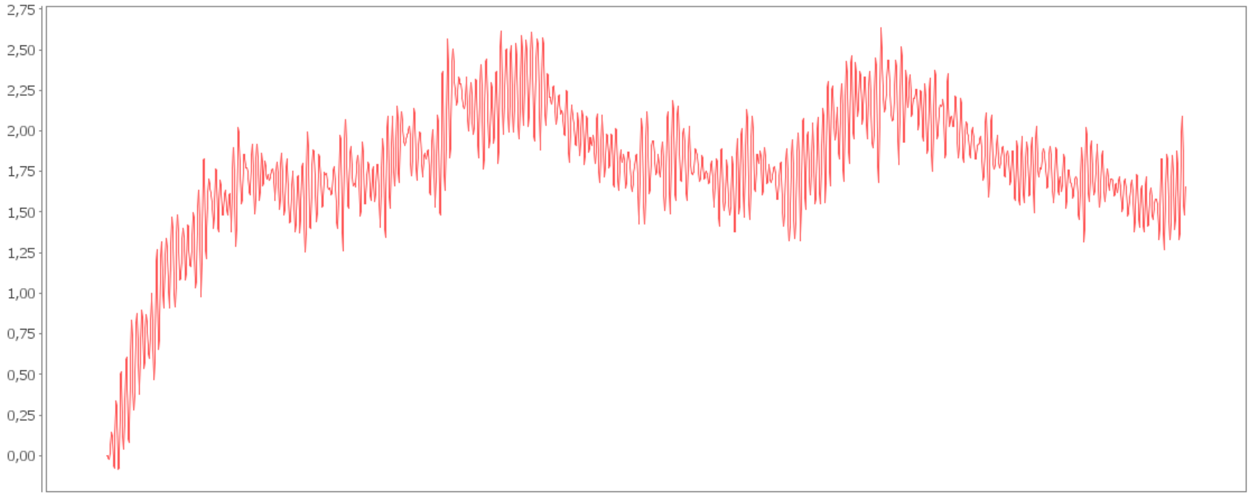

Figure 36 Response in irregular waves. Displacement *y*- direction. Length of time series is 13.125 minutes.

As seen by comparing Figure 36 to Figure 34 the maximum drift is much lower in the case with the irregular seas. The maximum displacement is almost 4 times larger in the case of the design wave. This is plausible. The variance in a spectrum is

$$\sigma^2 = \int_0^\infty S(\omega)d\omega = \frac{1}{16}H_s^{\ 2}$$

Equation 40

$$\sigma^2 = \frac{A^2}{2} = \frac{H^2}{8} = \frac{1.9^2}{8} H_s^{\ 2} = 7.22 \frac{H_s^{\ 2}}{16}$$

Equation 41

By comparing the variance it is seen that the variance of the wave train is approximately 7 times larger when analysing with a design wave than for irregular seas. As the drift force is proportional to the variance, the response to drift force can be expected to be 7 times larger than a realistic maximum drift force in cases where the mass of the system is such that several wave cycles are necessary to obtain the full amplitude for the response. This is the case in this example.

As seen by Figure 36 to Figure 34 the average response in the case of irregular seas are in this case larger than 1/7 of the response in a regular sea with wave height = 1.9 * Hs. It should be noted that the fact that AquaSim uses the instant position of coordinates to obtain the hydrodynamic response instead of the mean position. This will introduce a skewness in the average 1st order response. This is shown in Figure 37.

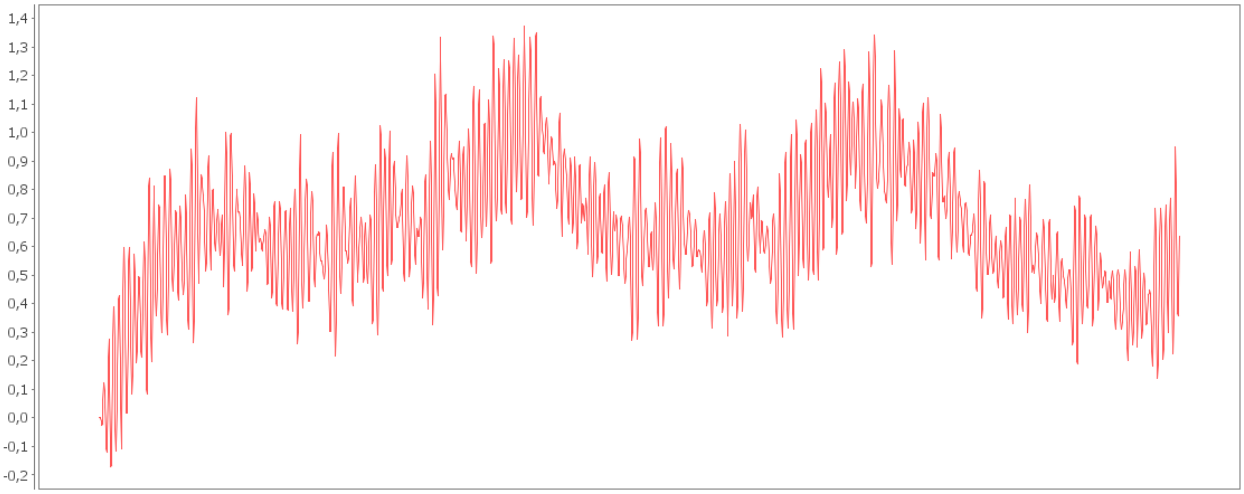

Figure 37 Analysis in same irregular sea as Figure 36 but not including the 2nd order wave drift force as formulated in AquaSim.

As seen by comparing Figure 37 to Figure 36 and further to Figure 34 it is seen that the difference in response due to drift force is in the range of 7 between irregular seas and regular seas.

This shows that

- AquaSim can be used to calculate drift forces.
- One should take care combining drift forces and design wave as the response to drift force may be largely overrated.

The skewness in the 1st order response is a function of the flexibility AquaSim have to account for large motions and rotations. AquaSim uses the instant position of the object as alternative to using the average position for the hydrodynamic loads. AquaSim may however introduce averaging of the wave period effect for this calculation in case that is a desired. The order of quality of the results will be the same as both alternatives are within 1st order accuracy.

3.3.6. Analysis of a typical barge case

A typical barge with mooring is shown in Figure 38.

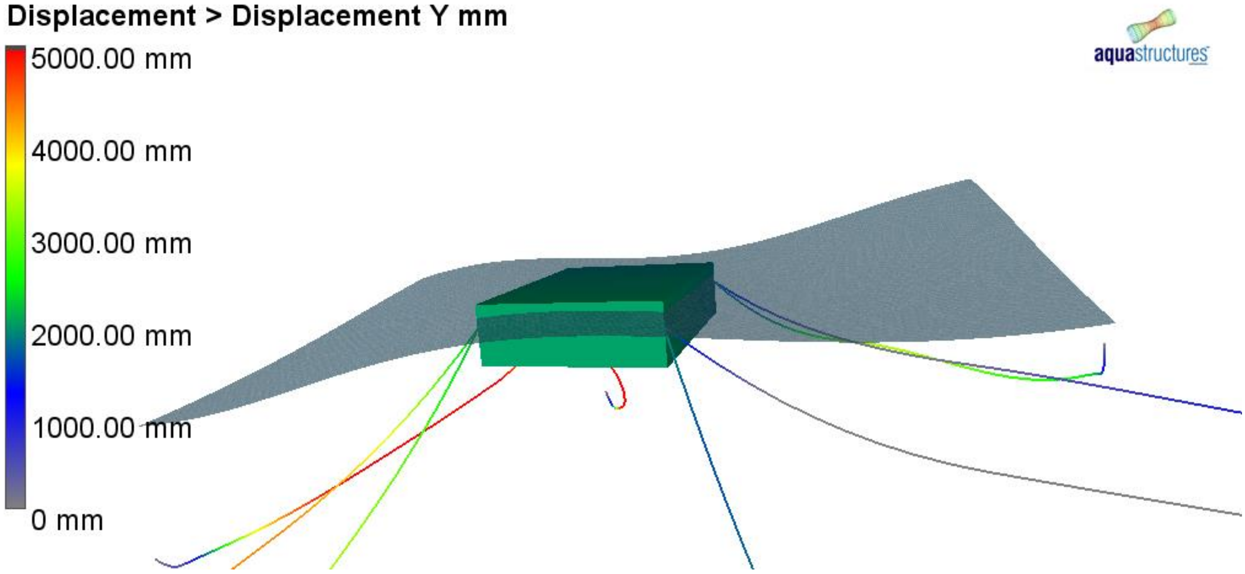

Figure 38 Analysed barge

The main particulars of the barge are given in Table 8 and key data for the mooring system is given in Figure 39.

Table 8 Main particulars barge and waves

Parameter	Value
Vessel	
Length [m]	22.5
Width [m]	17.6
Depth [m]	3.54
Deplacement [m2]	1401.84
Weight [kg]	1431000
Environment	
Hs	2
Hmax	3.8
Wave period (T, Tz)	6.4
Current velocity y [m/s]	0.4
Current velocity z [m/s]	-0.4

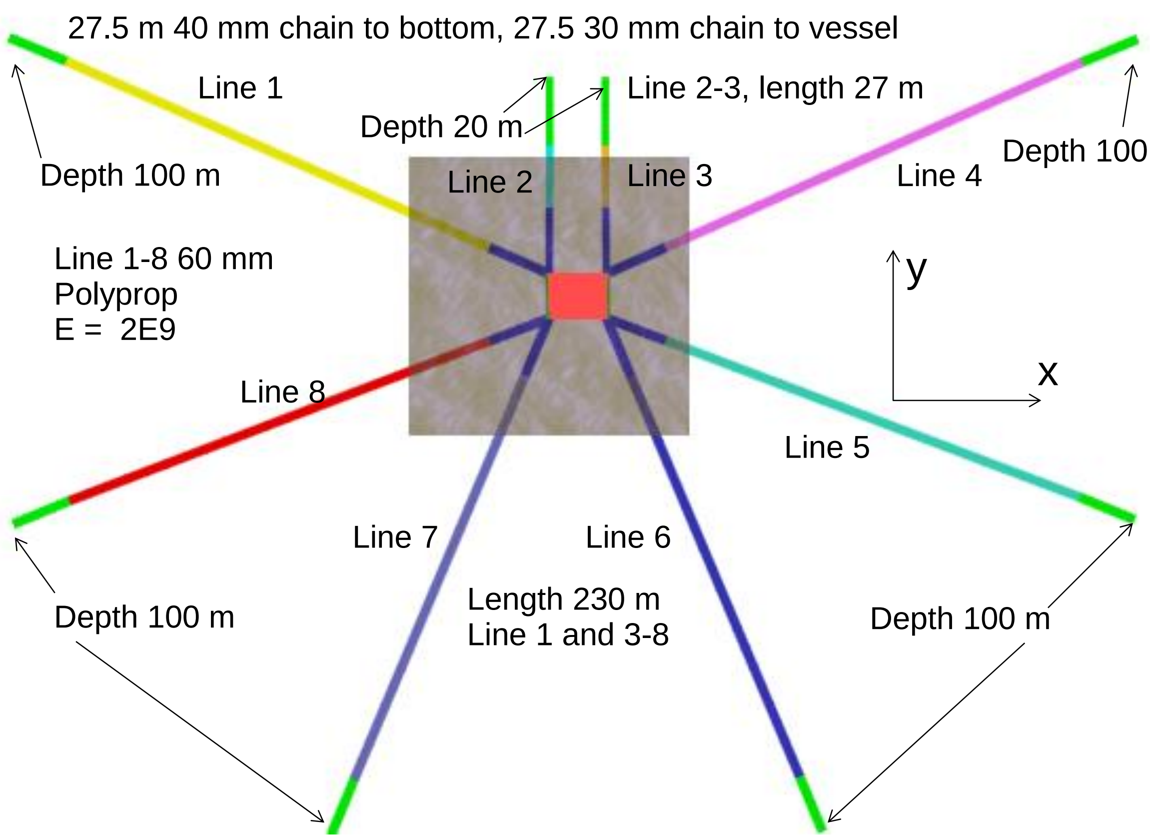

Figure 39 Key data mooring system

Figure 40 shows response in terms of displacement of the vessel in the y- direction.

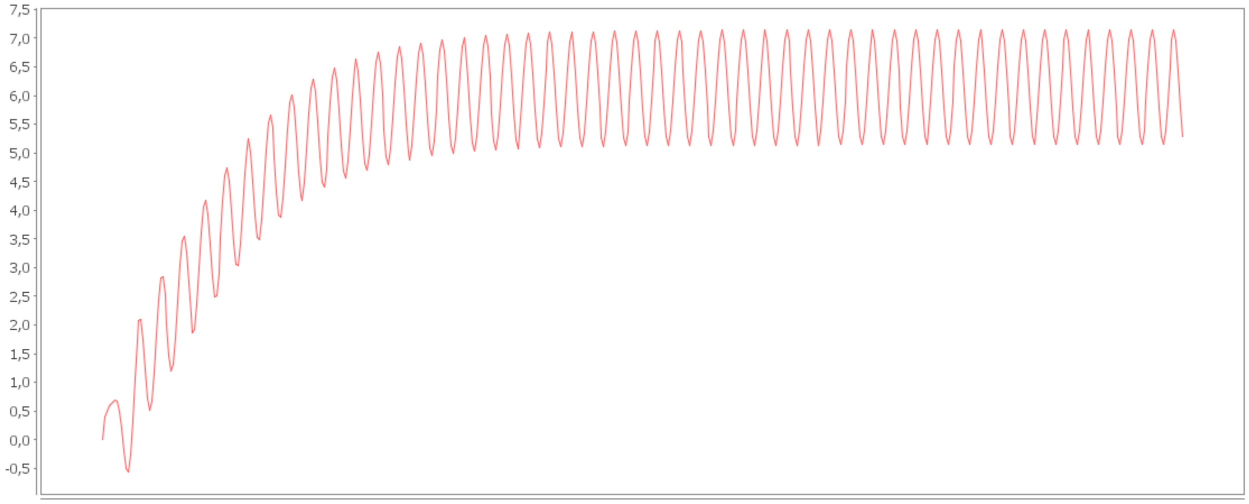

Figure 40 Response regular waves. Displacement in the y- direction. 5.3 minutes simulation time.

Figure 41 shows axial force in mooring line 7. As seen from Figure 41and Figure 40 the mooring load in line 7 is in proportion to the displacement which is logical. The analysis shown in Figure 41and Figure 40 has been analysed with a sinusoidal wave with wave height 3.8 meters.

Figure 41 Axial force in mooring line 7. 5.3 minutes simulation time.

Figure 42 and Figure 43 shows results for analysis in irregular seas with Jonswap spectrum and Hs = 2.0 meters.

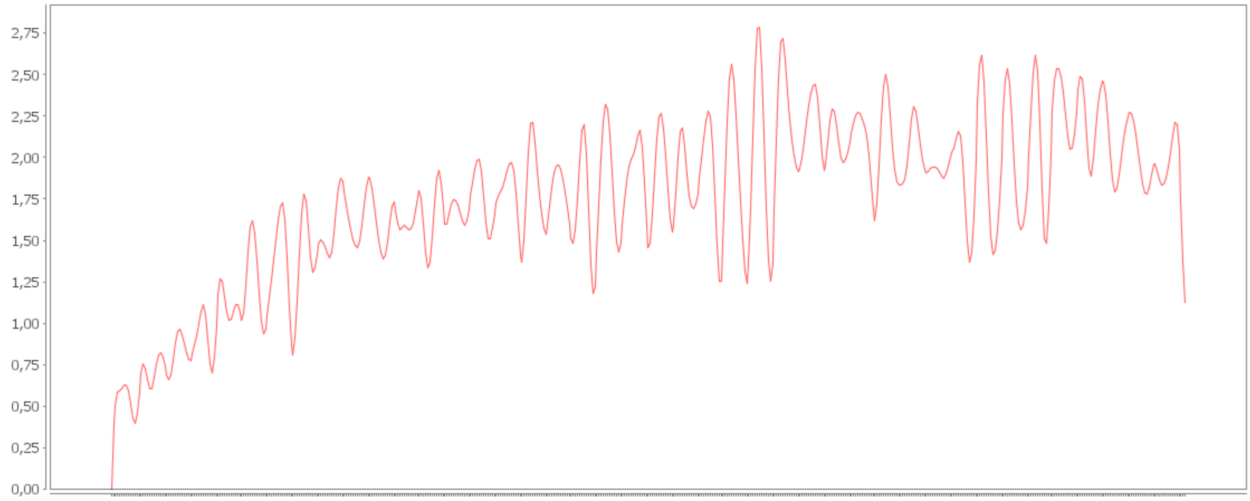

Figure 42 Response irregular waves. Displacement in the y- direction. 5.3 minutes simulation time.

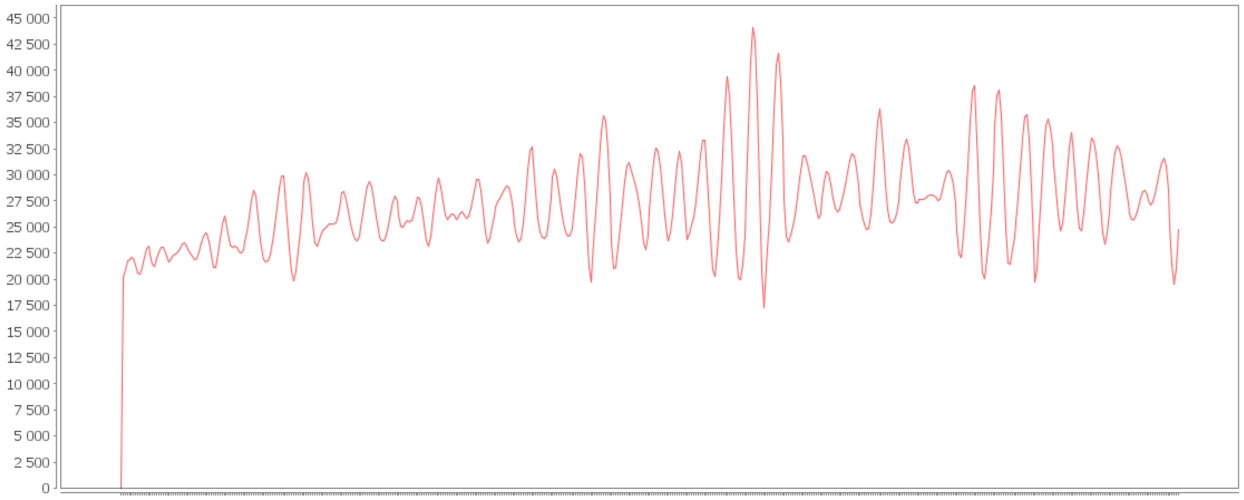

Figure 43 Axial force in mooring line, irregular wave

Comparing Figure 43 and Figure 42 to Figure 41and Figure 40 it is seen that displacements are in proportion with forces (which it should be). It is seen that the 3rd wave of the design wave analysis picks up the maximum value seen in irregular seas both for displacement and load in line 7.

Figure 44 shows a case where a conservative load along the y- axis has been introduced to the barge. No waves or currents are applied and static equilibrium is established. Displacement have been calculated for an increasing load an also the tangential stiffness have derived from the relation between displacement and force. The non-linear characteristic of the tangential stiffness is observed.

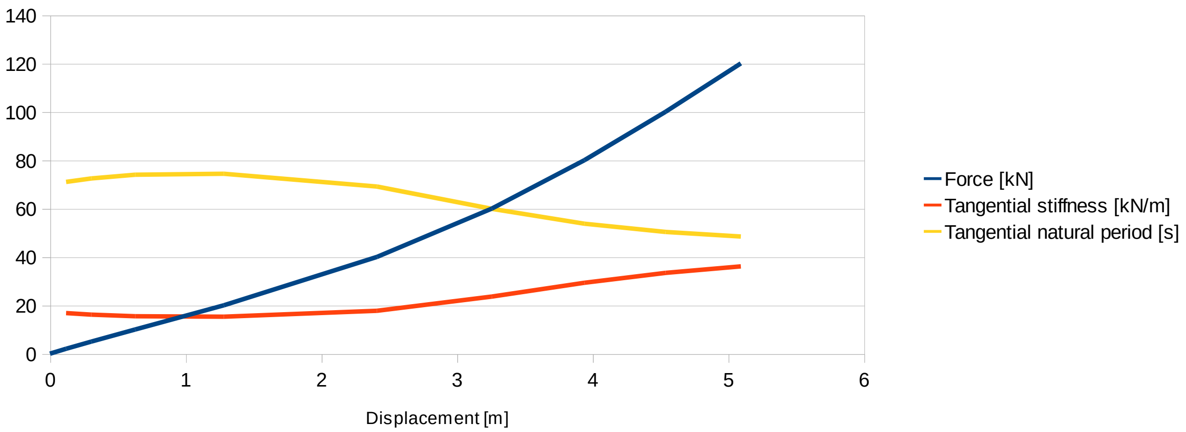

Figure 44 Force and tangential stiffness for mooring system.

As seen from Figure 44 the tangential stiffness varies this means also that the natural period (eigenperiod) varies as also shown in the figure.

Based on the analysis carried out in this case, the following can be concluded:

- Drift load formulation combined with a design wave approach will over-predict maximum response in mass controlled systems.
- This case did not show any significance of the natural period of the horizontal motion to the slow drift response.

Based on this it can be concluded that the main issue for drift, current and wind is to obtain a realistic and conservative mean displacement such that when the design wave hits, the system gets the appropriate total response. The stiffness of the system should be investigated as rate of change of stiffness to the system is critical to load impact effects.

3.3.7. Hydrodynamic loading for wave directions longitudinal to the vessel

Consider the case seen in Figure 39. Consider waves along the *x*- axis in this figure. Normally a vessel is modelled with strip theory along the longitudinal (x- axis in this case) direction. This is shown schematically in Figure 31. In the barge case this means the ends are not included in the load formulation. In order to include the hydrodynamic loads for a load case with waves along the x- axis one should model the vessel with strip theory along the y- axis for the case see in Figure 39. In order both to handle oblique seas and not needing to model several analysis models there is a possibility to introduce hydrodynamic loads and drift loads to a model for both the ship side and ends one may use normal strip theory for one direction and 2D strip theory in the other direction. As an alternative one may use drag elements at the end using the drag load formulation there. This is a simplified approach which may be good enough. One should also note that real cases consist of both waves and current whereas hydrodynamic theory only considers waves such that current need to be added anyway.

In order to discuss the magnitude of these loadings, consider a case where the end wall is large relative to wave length such that most of the incident wave is reflected. Assuming most of the wave is reflected, the hydrodynamic force can be derived as

$$\vec{F}_{FK+D} = -\iint_{Sw} p\vec{n}ds = \iint_{Sw} 2\rho g\zeta e^{kz}\sin(\omega t - kx)\vec{n}ds \approx DB\frac{2\rho g\zeta}{k}\sin(\omega t - kx)$$

Equation 42

Where D is the draught and B is the breath of the wall sided side of the vessel. This is the 1st order force. Only the ship side upstream relative to the waves are considered as an assumption for this consideration is that most of the incident wave is reflected and only a negligible part is transmitted.

From Morisons equation, for and end section with 0 volume and neglecting vessel velocity, the drag force will be:

$$F_{Drag} = \frac{1}{2}\rho C_d A(u)|u| = DB\frac{\rho C_d \omega^2\zeta^2}{2}\sin(\omega t - kx)|\sin(\omega t - kx)|$$

Equation 43

Comparing the amplitudes of Equation 41 and Equation 42 (and with the assumption of deep water):

$$\frac{F_{FK+D}}{F_{Drag}} = \frac{DB\frac{2\rho g\zeta}{k}}{DB\frac{\rho C_d \omega^2 \zeta^2}{2}} = \frac{\frac{2\rho g^2 \zeta}{\omega^2}}{\frac{\rho C_d \omega^2 \zeta^2}{2}} = \frac{4g^2}{C_d \omega^4 \zeta} = \frac{4g^2}{C_d k^2 g^2 \zeta} = \frac{4}{C_d k^2 \zeta}$$

Equation 44

Having a wave with steepness 1/7 this can further be written as

$$\frac{F_{FK+D}}{F_{Drag}} = \frac{4}{C_d k^2 \zeta} = \frac{4}{C_d \frac{(2\pi)^2}{\lambda^2} \zeta} = \frac{\lambda^2}{C_d \pi^2 \zeta} = \frac{14\lambda}{C_d \pi^2} \approx \lambda = 7h$$

Equation 45

with a C_d of approximately 1.5. For normal cases and values this means forces into the flat "bow" or stern of the vessel can be underestimated.

Consider also the drift for

$$\frac{F_2}{F_{Drag}} = \frac{DB\rho g \zeta^2}{DB\frac{\rho C_d \omega^2 \zeta^2}{2}} = \frac{g}{\frac{C_d \omega^2}{2}} = \frac{2g}{C_d k g} = \frac{2}{C_d k} = \frac{\lambda}{C_d \pi} \approx 4h$$

Equation 46

This shows that also drift load is of the same magnitude.

Consider the 5 load variation cases for the vessel with respect to waves and current in the *x*-direction:

1. Froude Krylov and diffraction forces accounted for by strip theory onto end section according to description in Section 2.5. In addition also drift forces are introduced and drag loads of the mean wetted hull where the drag area equals the half of the mean wetted hull at both sides are included.
2. Froude Krylov and diffraction forces accounted for by strip theory onto end section according to description in Section 2.5. In addition also drift forces are introduced
3. Froude Krylov and diffraction forces accounted for by strip theory onto end section according to description in Section 2.5. No drift forces
4. No Froude Krylov and no drag on the ends
5. No Froude Krylov and draft area as drag area on the ends.

Results for waves along the positive x-axis and with environmental data as given in Table 9 is shown in Figure 45:

Table 9 Environment

Environment	
H	3.8
Wave period (T)	6.4
Wave direction [Deg]	0
Current velocity x [m/s]	0.4

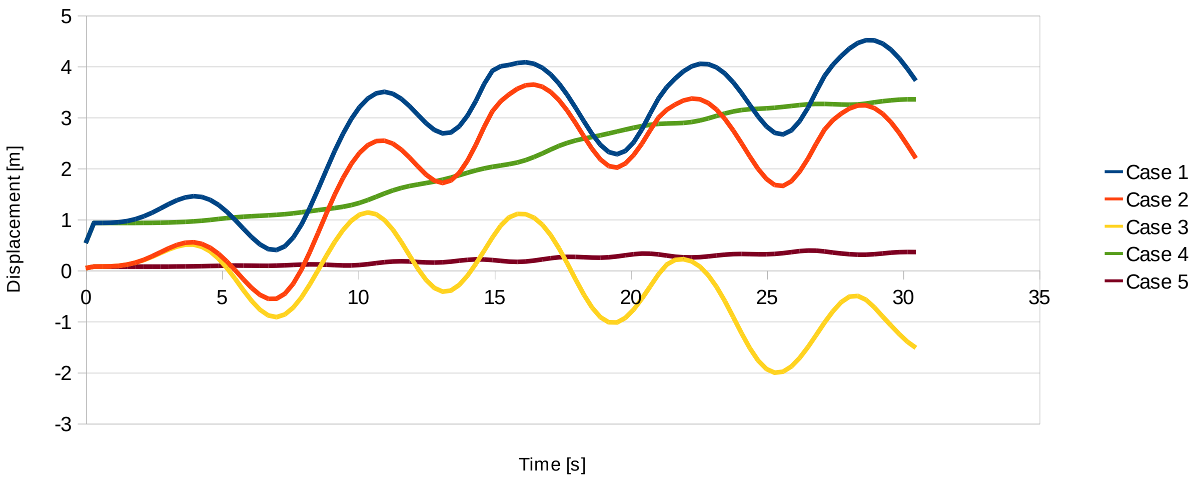

Figure 45 Results comparison 4 cases with different load formulation.

As seen from Figure 45 results depend on load formulation.

1st order hydrodynamic theory does not account for current. In the 2nd order drift term this effect is included. This is why case 3 and 5 does not pick up the overall average force and motion in the x- direction.

Including forces to the bow and stern by the Morison equation using the mean wetted hull as means drag area should be used on wetted surfaces to be certain this is included in results. As seen from Figure 45 using a drag formulation based on the mean wetted hull picks up the motion rather well. This means if the vessel is not too wide compared to length this could be a sufficient alternative.

The wave drift formulation accounts for both current and waves. As seen from the results using this without accounting for the 1st order drag area gives results that look fine.

In case 1 all effects is included and it will in practice means it is overlapping and conservative. This is then the safest approach with respect to design.

4. CONCLUSIONS

Several analysis have been carried out validating the ability of AquaSim to calculate response of large mass moored vessels.
The hydrodynamic loads to the vessel is calculated integrated with motion and forces in the mooring system such that nonlinear response is accounted for. This document shows how forces from hydrodynamic loads are introduced to AquaSim. The document describes the options AquaSim users have to account for the load components which are of importance for response.

Relevant natural periods of systems and loads occurring should be carefully evaluated by the analyst to ensure that all relevant load-response-effects are considered.

The importance of velocity and mass of floating object, stiffness properties of moorings are shown both through analysis and analytical formulae. Analysis using AquaSim compares well with analytical formulae for cases where analytic formula can be established.

AquaSim shows good versatility.

5. REFERENCES

Aquastructures (2012) “Verification and benchmarking of AquaSim, a softwaretool for safety simulation of flexible offshore facilities exposed to environmental and operational loads“, Aquastructures report 2012-1755-1.

Aquastructures (2013) “The AquaSim Package user manual” Aquastructures report TR-30000-2049-1.

Faltinsen, Odd M. (1990) “Sea loads on ships and offshore structures.” Cambridge university press ISBN 0 521 37285 2

Fathi, D. (1996). VERES Version 2.0 – User’s manual. Marintek, 7034 Trondheim, Norway.

Morison, J, M.O’Brien, J. Johnson and S. Shaff (1950). “The forces Exerted by surface waves on piles.” Petrolium Trans. 189, 149-1.

- o0o -