Shell elements in AquaSim, case studies for validation

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Summary:
Analysis has been carried out for 2 cases where response from shell elements is compared to analytic formulae or beam elements. It is shown how additional response effects are covered by shell elements, and the effect of shear locking of shell elements is shown and discussed.

Based on the analysis carried out in this document it is concluded that the shell element behaves as expected and can be a useful tool for analysis

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1 Introduction

Shell elements is one option for elements to use in AquaSim (ref /1/ and /2/). This document presents a comparison of shell elements to analytic results and beam models.

2 Case study 1 L beam

A simplified analysis case has been established as shown in Figure 1.

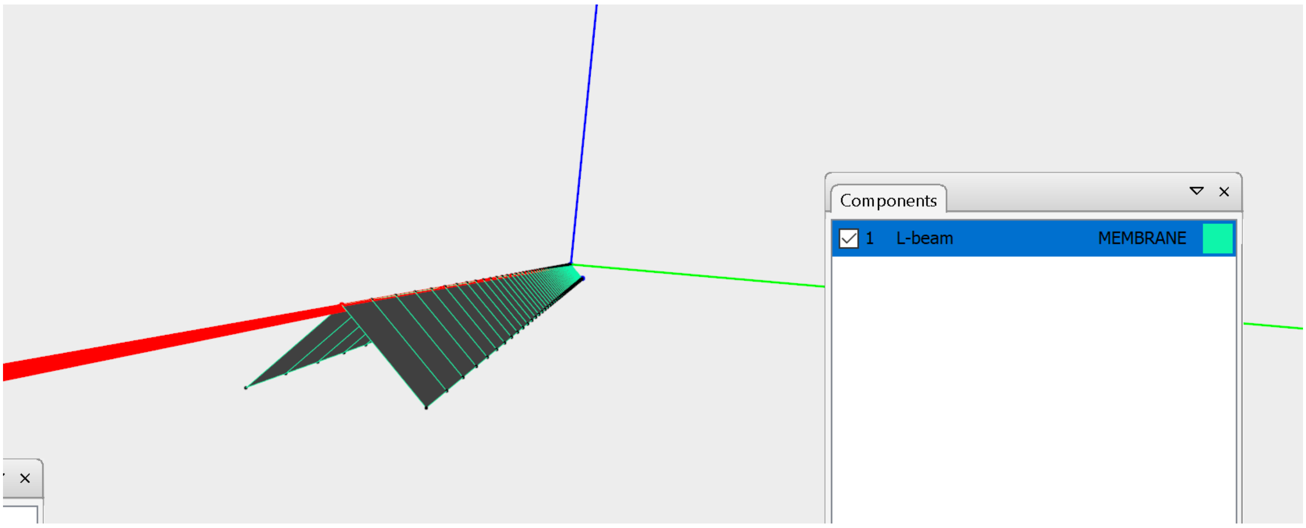

Figure 1 Shell elements

The structural data for the case study is given in Table 1.

Table 1 Structural data case study

Elastic modulus [N/m^2]	900000000
Height of each side [m]	0.2
Thickness [m]	0.01
Length [m]	10
Cross sectional area [m^2]	0.004
Angle of arms [DEG]	45

A beam model has been established for the same case as shown in Figure 2.

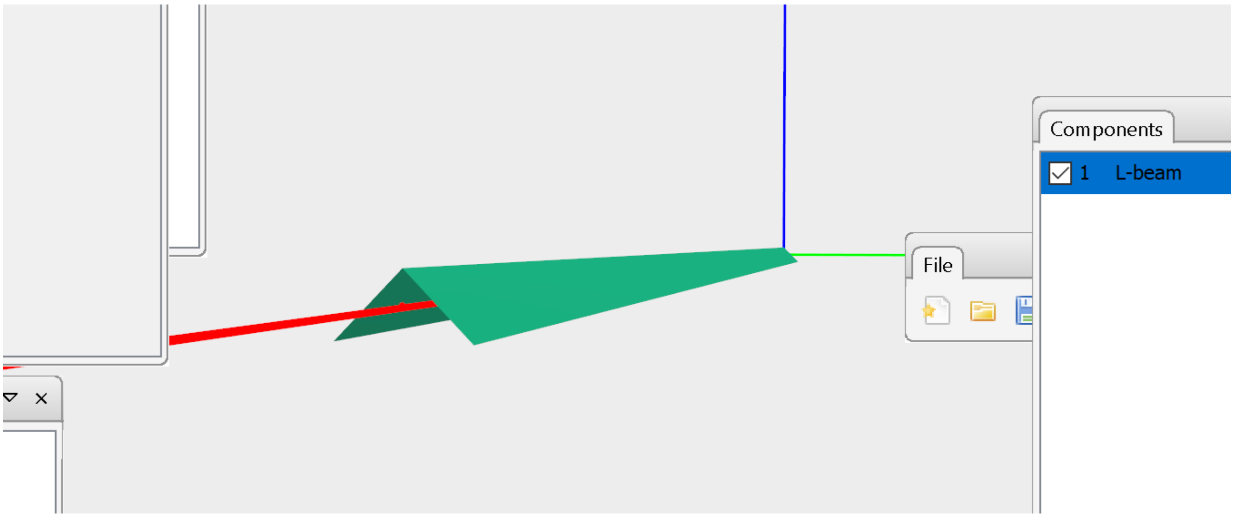

Figure 2 Beam model.

Figure 3 shows displacement at the end as a function of load.

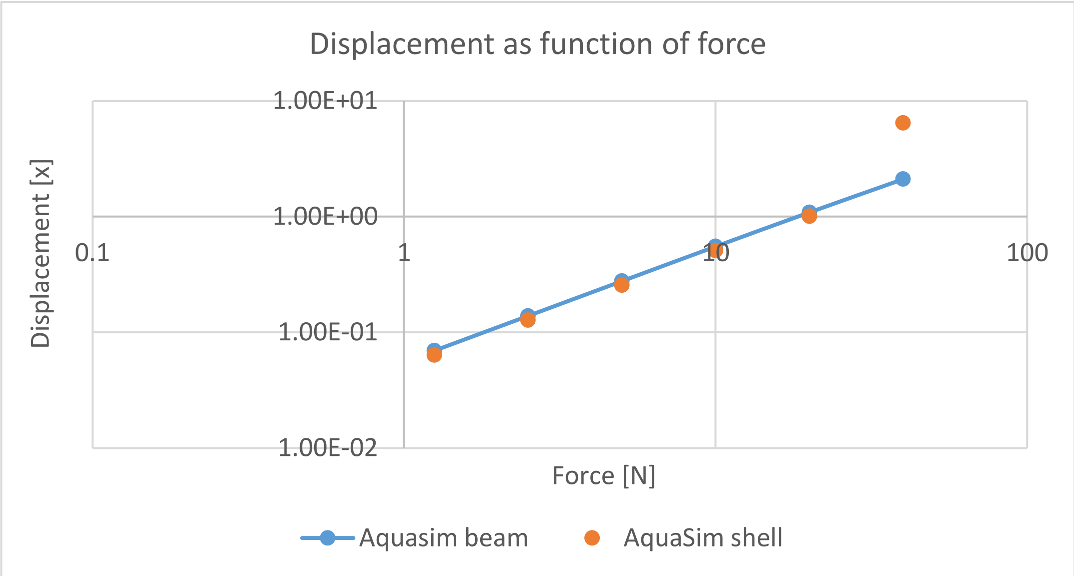

Figure 3 Displacement as function of force, beam model and shell model

Table 2 shows the results in Figure 3 in numbers.

Table 2 Results, numerical values results Figure 3.

Force	Aquasim beam	AquaSim shell	Difference %
1.25	6.93E-02	6.37E-02	-8.00 %
2.5	1.39E-01	1.28E-01	-7.94 %
5	2.77E-01	2.55E-01	-7.75 %
10	5.52E-01	5.10E-01	-7.73 %
20	1.09E+00	1.01	-7.34 %
40	2.11E+00	6.47	206.00 %

As seen from the results in Table 2, the shell element model seems to give approx. 8% less displacement at the end than a beam mode. This is further discussed in succeeding sections. Transverse effects are included to the shell element. Which means this model will include local buckling in the beam by change of shape by the flanges of the L beam, being bent upwards reducing the moment of inertia. This effect is not included in the beam model as the beam model will have a constant area moment of inertia throughout the analysis. The effect of the shape change leading to reduced area moment of inertia is seen in Figure 4.

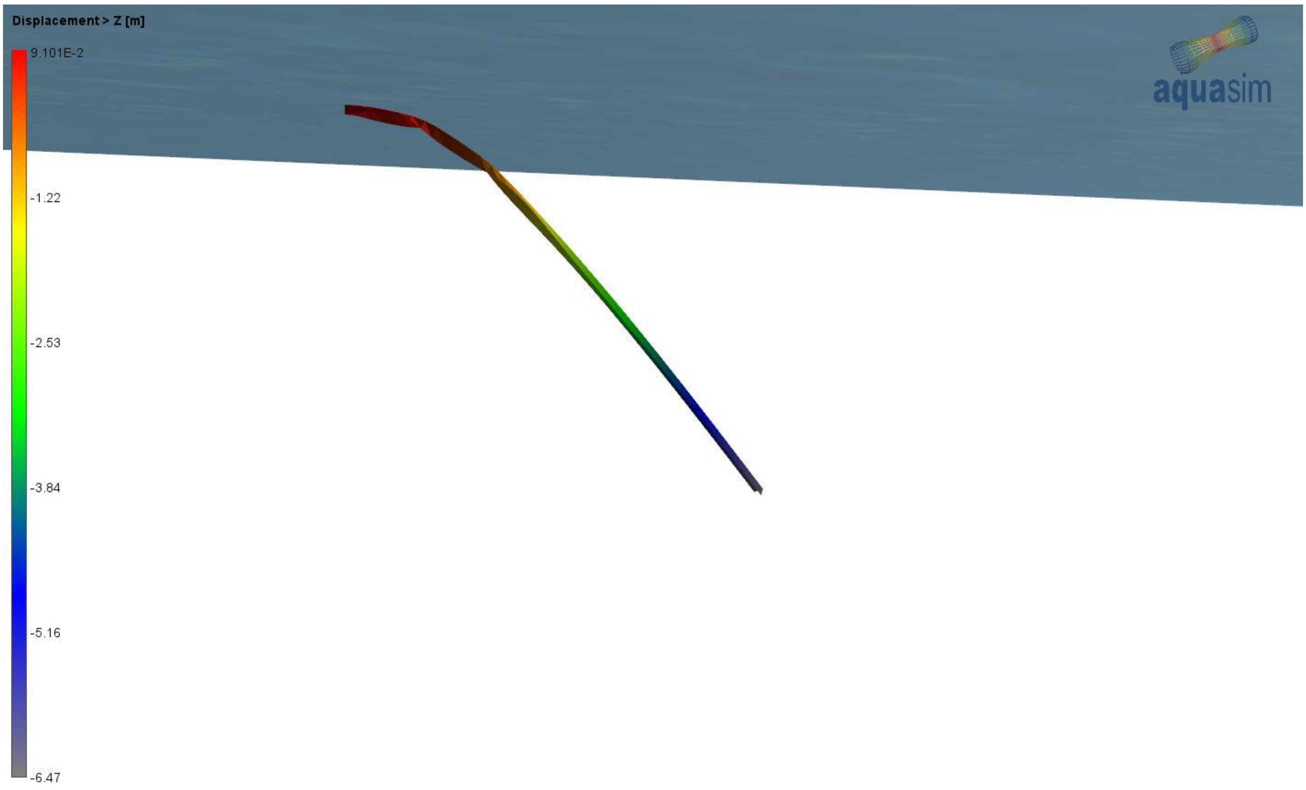

Figure 4 Displacement of shell model. Buckling pattern can be observed.

This case then illustrates both that shell elements may be a bit stiff in certain degrees of freedom (shear locking) and that shell elements will include response pattern which are not included in a beam element.

2.1.1 Refined model

In order to investigate this a refined model with 900 elements as shown in Figure 5 has been established.

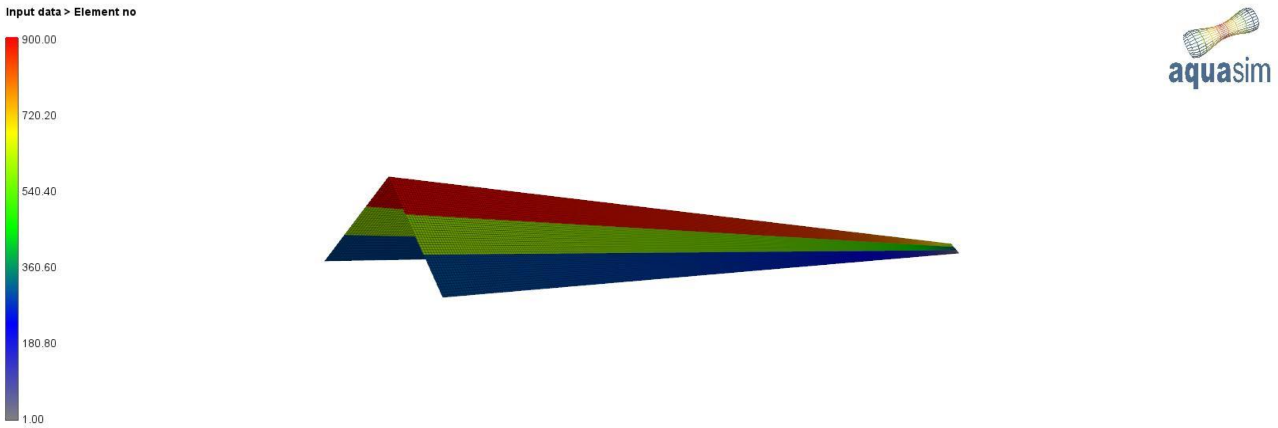

Figure 5 Elements in refined model of L-beam. 150 along the length of the beam and 3 on each flange.

The refined model is denoted «Aquasim shell refined, and results are seen in Figure 6 and Table 3

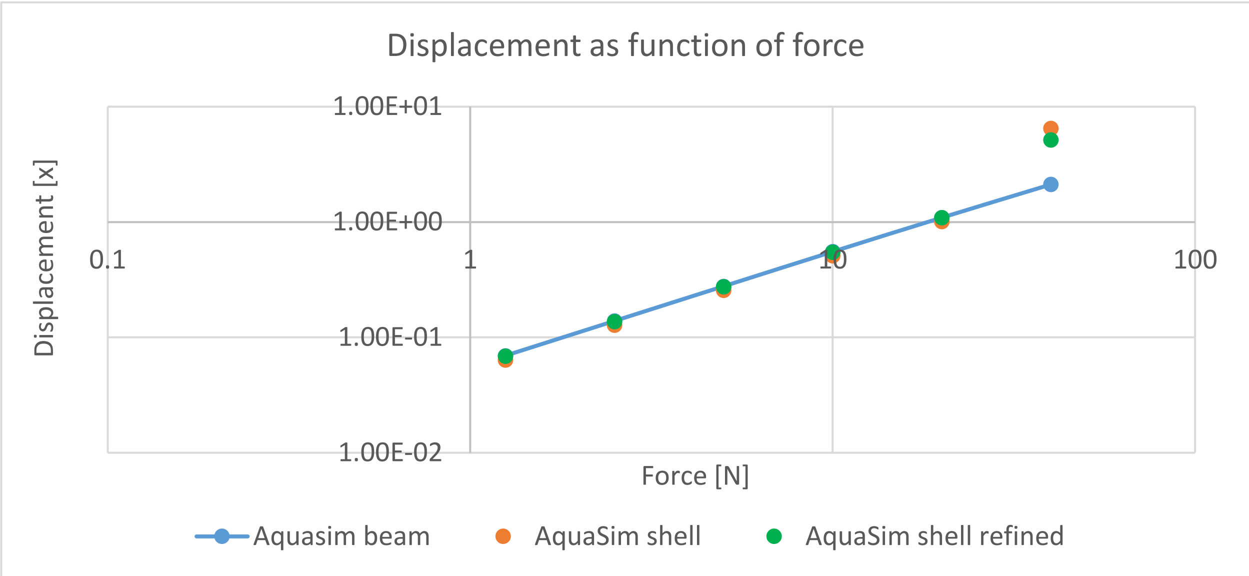

Figure 6 Comparison o fresults from analysis models.

Table 3

Force	Aquasim beam	AquaSim shell refined	Difference %
1.25	6.93E-02	6.87E-02	-0.78 %
2.5	1.39E-01	1.37E-01	-0.94 %
5	2.77E-01	2.74E-01	-1.08 %
10	5.52E-01	5.47E-01	-1.03 %
20	1.09E+00	1.09E+00	0.00 %
40	2.11E+00	5.16E+00	144.04 %

As seen in Figure 6 and Table 3 results between beam model and shell model compares much better for the refined model. This is further discussed in subsequent sections.

2.2 I beam case study

Figure 7 shows a cantilever beam modelled as a shell element. The beam has been modelled with the shell elements in the x-z – plane and the thickness in the y- direction. Loads to the endpoint has been varied. The analysis model has 50 elements all 0.2x0.2 meters in size.

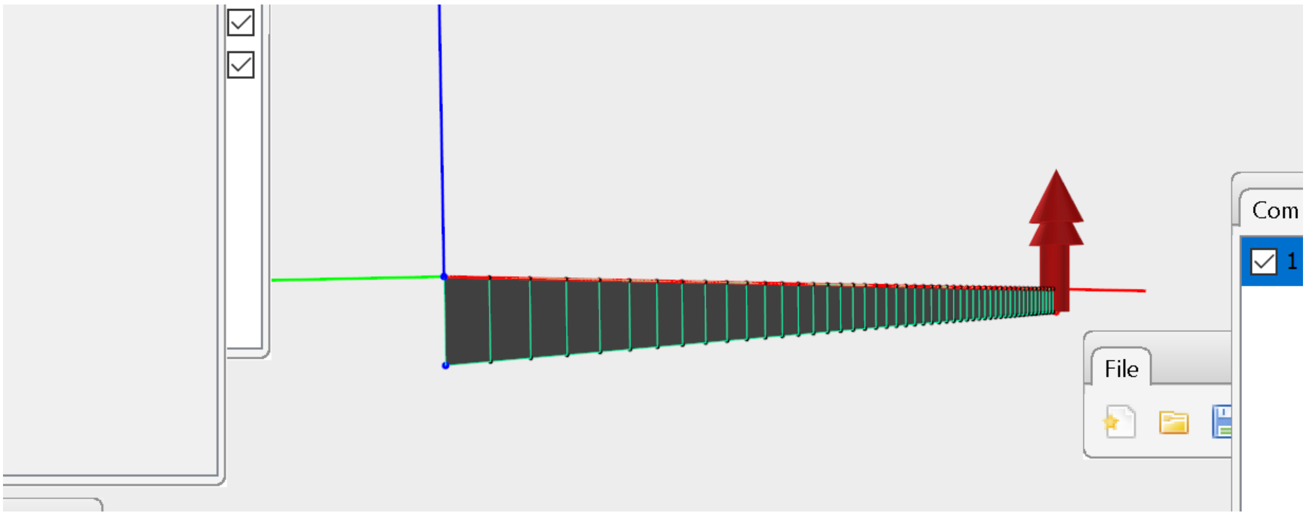

Figure 7 Cantilever I-beam modelled with shell elements. Node loads at the end have been varied. 50 elements.

The structural data for the beam is given in Table 4.

Table 4 Data for the model

Parameter	Abbreviation	Value
Elastic modulus [N/m2]	E	9.00E+08
Height [m]	H	0.2
With [m]	W	0.1
Length [m]	L	10

Analytic results have been found by applying the beam equation for tip displacement of a cantilever beam:

$$k = \frac{3EI}{L^3}$$

Equation 1

Where

$$I = \frac{WH^3}{12}$$

Equation 2

Displacement, u is

$$u = \frac{F}{k}$$

Equation 3

The analytic results is then compared to numerical analysis.

2.2.1 Case 1 load in axial direction

Figure 8 shows displacements as function of axial load. As seen from the figure, results compare well with analytical results.

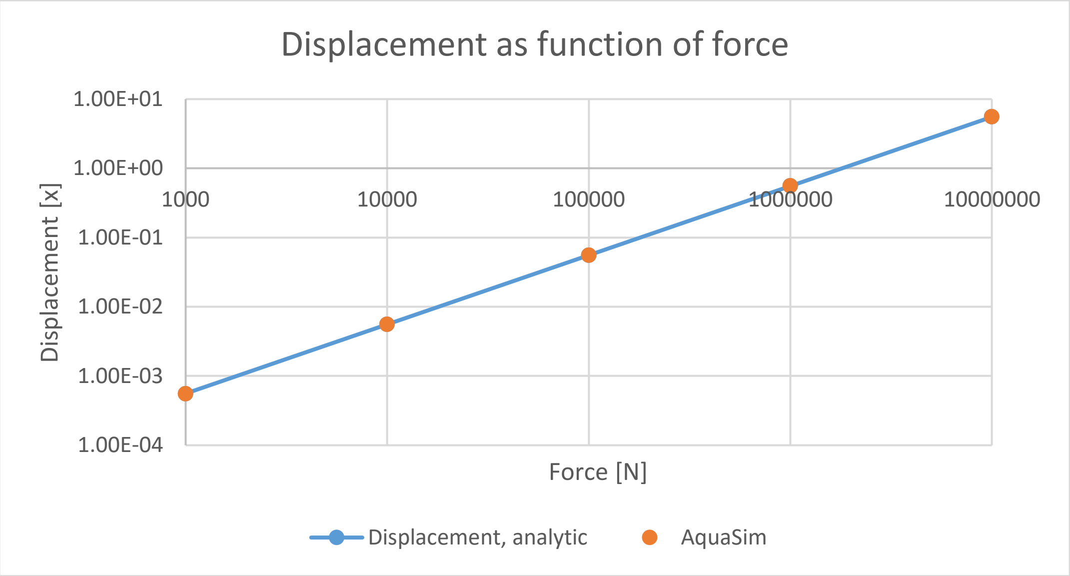

Figure 8 Displacements as function of axial load.

2.2.2 Case 2 load in thickness direction

In load case 2 a node load has been divided by the outer two nodes (upper and lower) in the y- direction. Results are compared to analytic formulae in Figure 9 and the same number are given in Table 5.

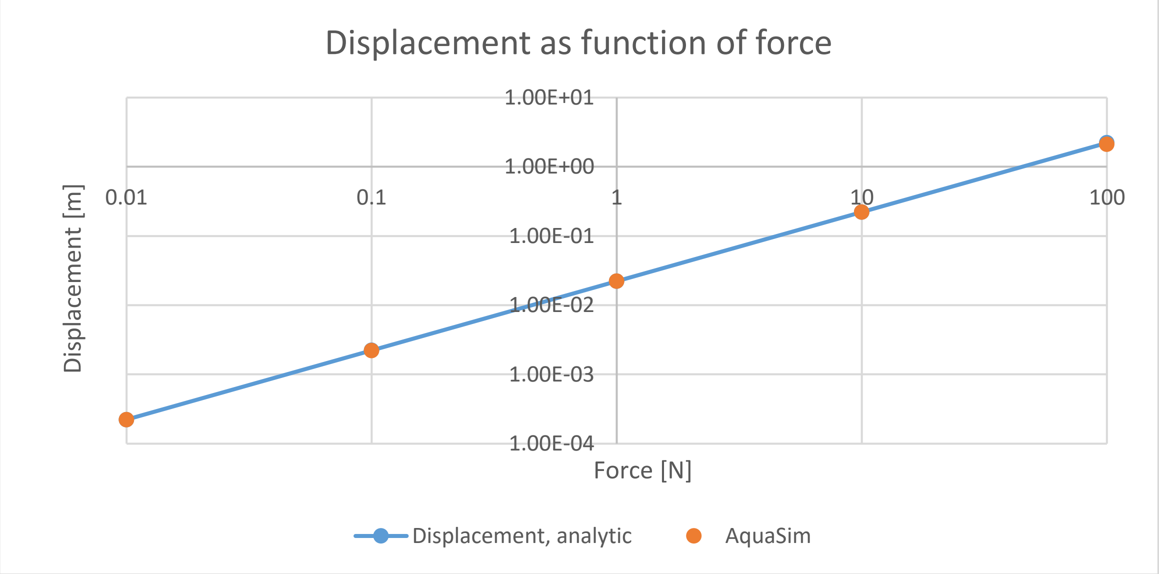

Figure 9 Displacement y- as a function of load

Table 5 Results, Figure 9.

Force	Displacement, analytic	AquaSim	Difference %
0.01	2.22E-04	2.21E-04	-0.68 %
0.1	2.22E-03	2.22E-03	-0.32 %
1	2.22E-02	2.22E-02	-0.28 %
10	2.22E-01	2.22E-01	-0.32 %
100	2.22E+00	2.113	-4.91 %

The results are comparable apart from the last result where the numerical results are lower. In this case, however, there is a nonlinearity as displacements are getting larger such that it is expected that the displacement is smaller for this case. This is further investigated for loads in z- direction.

2.2.3 Case 3 load in z- direction

Figure 10 and Table 6 shows displacements calculated in AquaSim compared to analytic formulae.

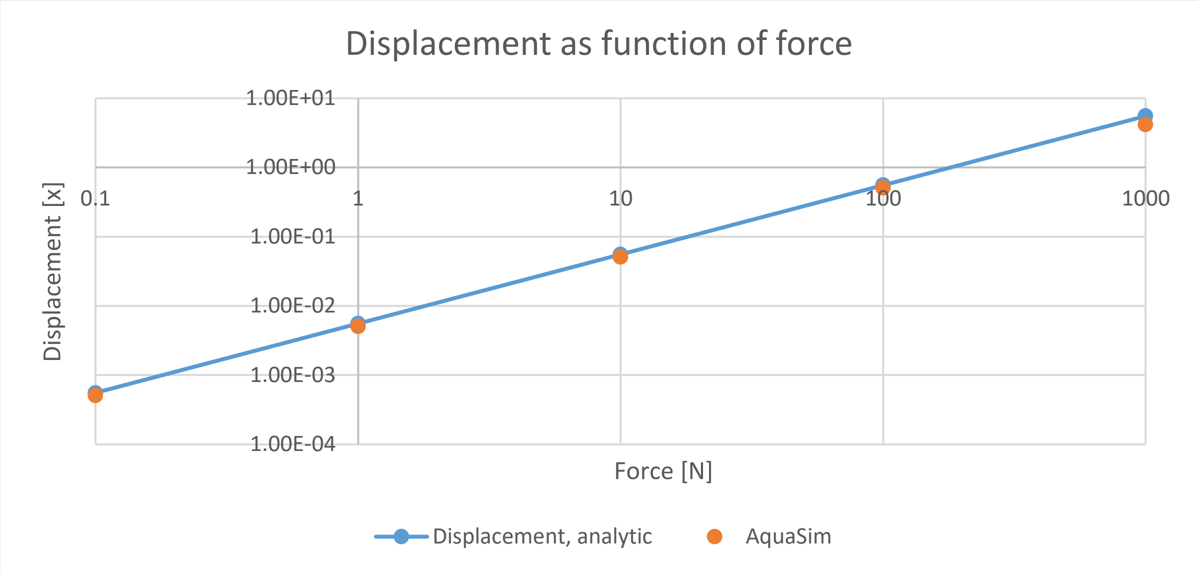

Figure 10 Aquasim analysis compared to analytic formulae

Table 6 Analysis compared to analytic formulae

Force	Displacement, analytic	AquaSim	Difference %
0.1	5.56E-04	5.07E-04	-8.79 %
1	5.56E-03	5.07E-03	-8.78 %
10	5.56E-02	5.07E-02	-8.78 %
100	0.56	0.51	-8.96 %
1000	5.56	4.17	-24.94 %

As seen from the results, the analytic formulae predict approximately 9 % larger displacements than the analytic formula. The stiffer response from the shell element is probably due to shear locking. Shear locking is an error that occurs in finite element analysis due to the linear nature of quadrilateral elements. The linear elements do not accurately model the curvature present in the actual material under bending, and a shear stress is introduced.

The effect of shear locking shall be smaller if more elements are introduced. In order to investigate further a refined analysis model was established as shown in Figure 11. This model has 9 times more elements than the original model.

2.2.3.1 Refined model

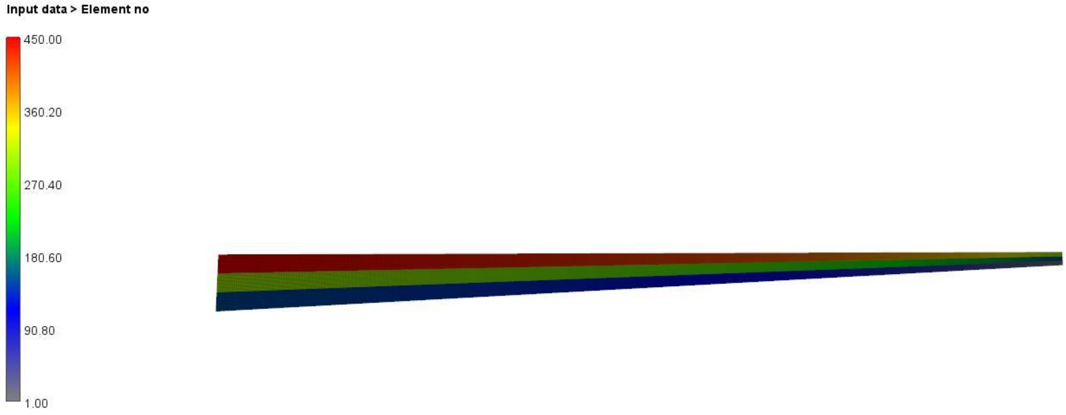

Figure 11 Refined analysis model with 450 elements. 150 elements in the axial direction times 3 elements over the height.

Results are given in Figure 12 and Table 7.

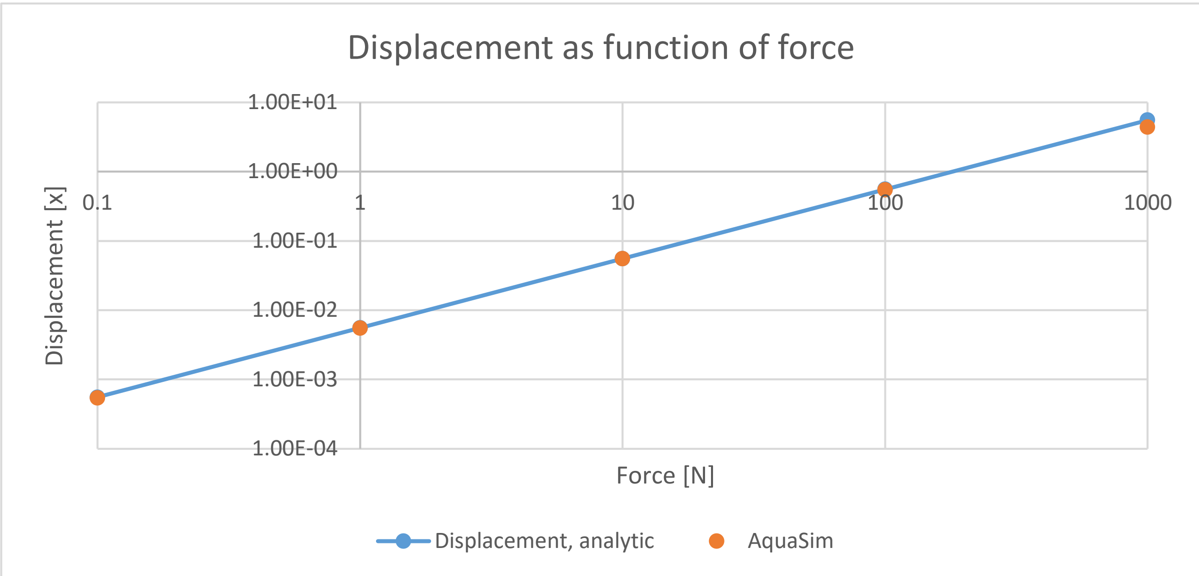

Figure 12 Comparison

Table 7 Comparison

Force	Displacement, analytic	AquaSim	Difference %
0.1	5.56E-04	5.43E-04	-2.30 %
1	5.56E-03	5.49E-03	-1.20 %
10	5.56E-02	5.52E-02	-0.60 %
100	0.56	0.55	-1.38 %
1000	5.56	4.40	-20.80 %

As seen from Table 7, results compare very well apart from the case with largest loads. In this case the deflection is seen in Figure 13. It is expected in this case that displacements are smaller since response is not linear anymore.

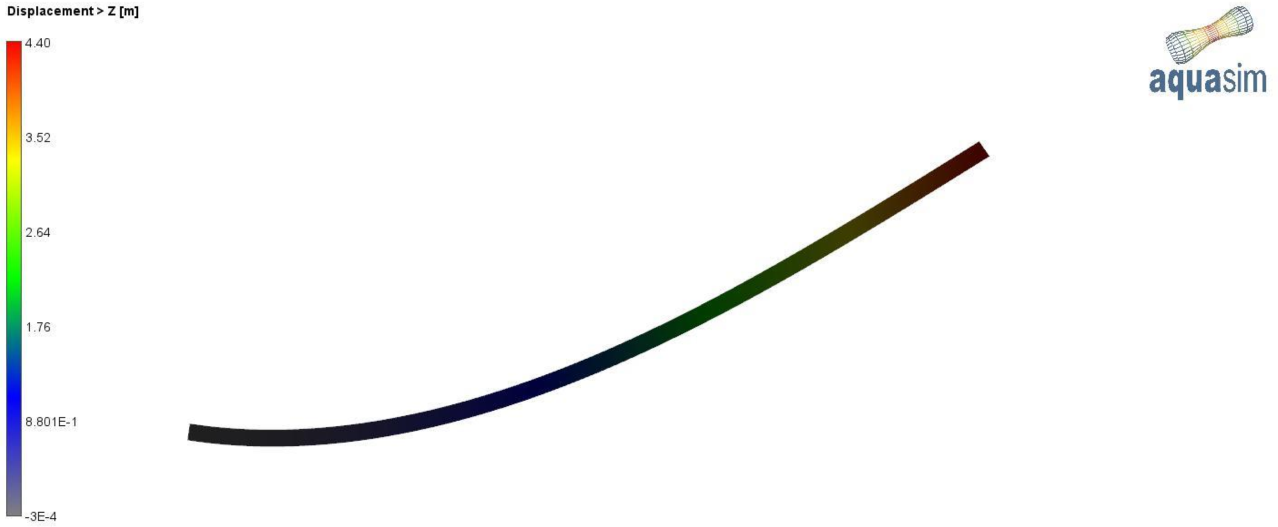

Figure 13

In order to validate the shell results for this case, a beam model was established of the same case, and the same load was applied. Results are seen in Figure 14, and as seen, results compare very well with less than 1% difference in tip motion (4.40 m vs 4.41 m).

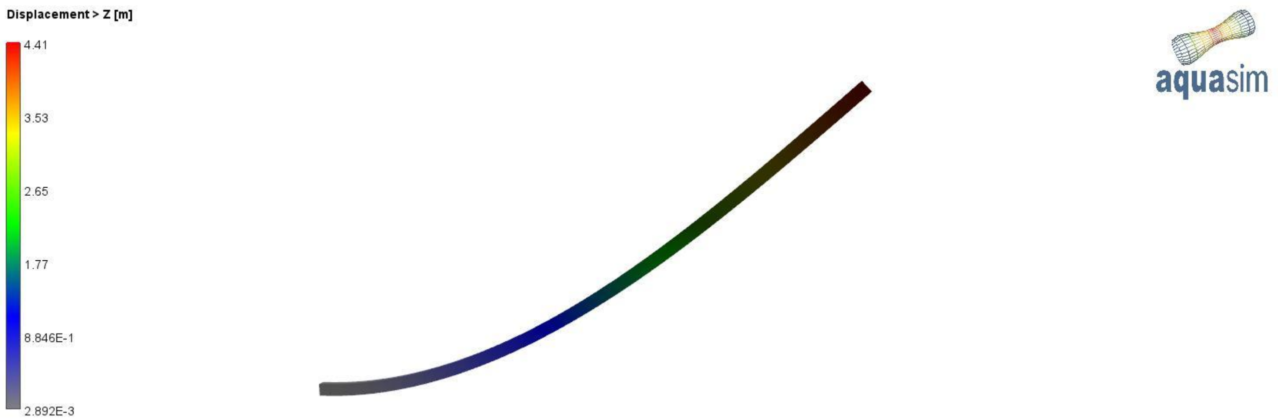

Figure 14 Beam model

A stress level comparison has been carried out for the refined shell model and the beam model. Results are shown in Figure 15. The max stress level in the beam model is 4% larger than the shell model. When assessing the distribution of Von Mises stress over the cross section, it looks like the shell elements provides a good presentation of the stress distribution over the cross section.

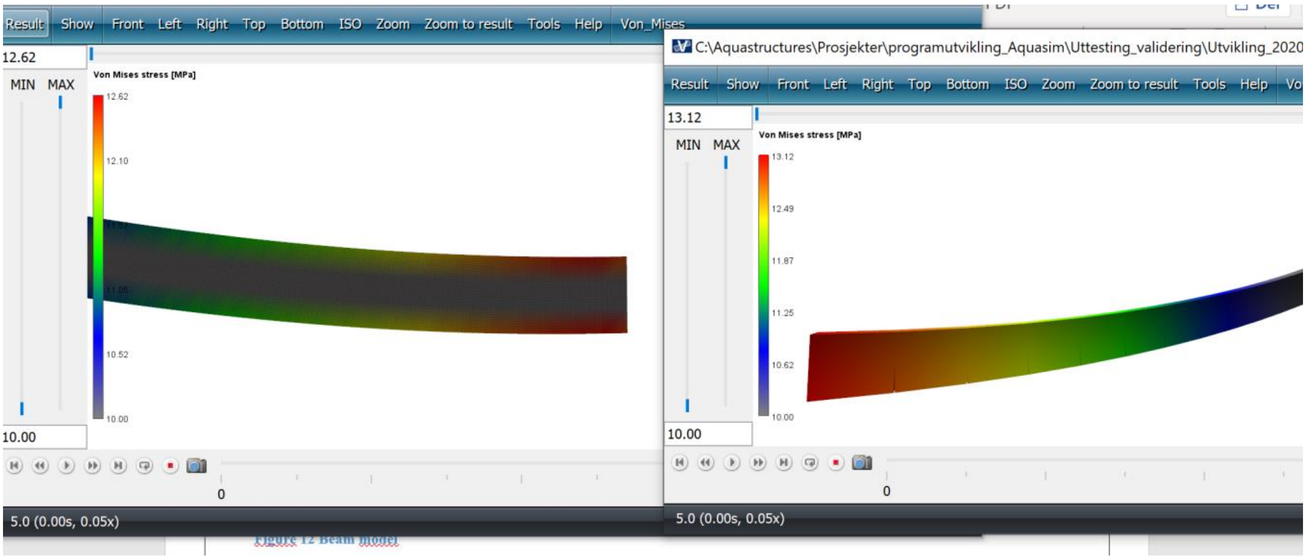

Figure 15 Von Mises stress comparison shell to beam elements

3 Conclusions

Based on the analysis carried out in this document it is concluded that the shell element behaves as expected and can be a useful tool for analysis. One should be aware of the shear locking effect fro shell elements and make sure that effect is not influencing results in an important way.

4 References

1. Aquastructures (2013) “The AquaSim Package user manual” Aquastructures report TR-30000-2049-1
2. Aquastructures (2012) “Verification and benchmarking of AquaSim, a softwaretool for safety simulation of flexible offshore facilities exposed to environmental and operational loads“, Aquastructures report 2012-1755-