Morison free plate load formulation to shell and membrane elements

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Revision 1

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Summary:
This report summarizes verification study of the load formulation ‘Morison free plate’. Numerical calculations have been compared to analytical formulae.

Shell- and tarp elements have been used in this study. Comparison of numerical and analytical results show good correspondence.

Revision 2:
Corrections to the description of C_a in Equation 1. Note (1) is added.

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1 Introduction

The load formulation “Morison free plate” has been introduced to AquaSim (e.g. ref /1/ and /2/). In this load formulation, loads are calculated by the Morison equation applying the cross flow principle to a membrane og shell element. This means there is no interaction effects between several elements on a structure.

2 Theoretical formulation

Consider a flat plate in water, as seen in Figure 1. N is the normal vector to the plane of the plate. Using Morison Equation and the cross flow principle, a force normal to the plane can be calculated as:

$$F_N = \underbrace{\frac{\rho_w C_d A}{2}(u_N - v_N)|u_N - v_N|}_{\text{Drag force}} + \underbrace{\rho_w A t \dot{u}_N}_{\text{Froude-Kriloff}} + \underbrace{\rho_w A C_a(\dot{u}_N - \dot{v}_N)}_{\text{Added mass and damping}}$$

Equation 1

Where

ρ_w is the density of the fluid

C_d is the drag coefficient

A is the area of the plate element.

u_N is the incident fluid velocity normal to the plate

v_N is the normal velocity of the plate

t is the thickness of the plate. In case plate is modelled as flexible tarp it is the equivalent thickness of twines giving the same volume when multiplied with the area A.

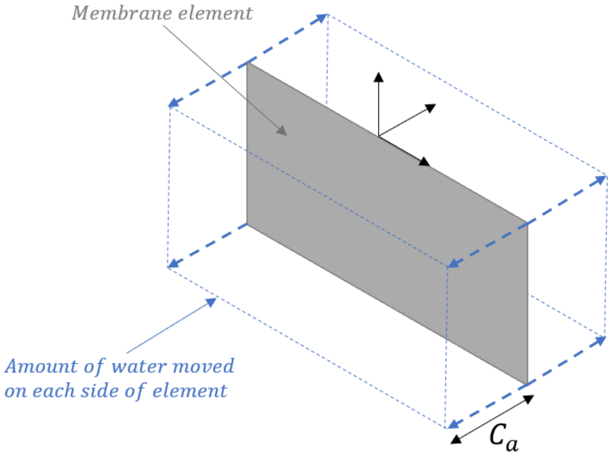
C_a is the added mass coefficient. Input value to AquaSim, unit is [m]. More information in Note (1).

$\dot{u}_N$ is the incident fluid acceleration normal to the plate.

$\dot{v}_N$ is the element acceleration the plate in the normal direction.

Note (1)

The added mass coefficient indicates the amount of water on each side of the element is moved due to the motion of the element, see figure below. Example: if C_a=1.0, then 1m of water on each side of the element is moved.



Forces in the tangential direction of the plate (uniform in all directions) are calculated as:

$$F_t = \frac{\rho_w C_t A}{2}(u_t - v_t)|u_N - v_N|$$

Equation 2

Where

C_t is the drag coefficient for tangential drag

u_t is the incident fluid velocity tangential to the plate

v_t is the tangential velocity of the plate

Case studies are presented to show the utilization and for verification. The direction of the force is in the direction of the in-plane vector $u_t - v_t$.

3 Case study 1, Simple 1 shell element

A simplified analysis case has been established as shown in Figure 1.

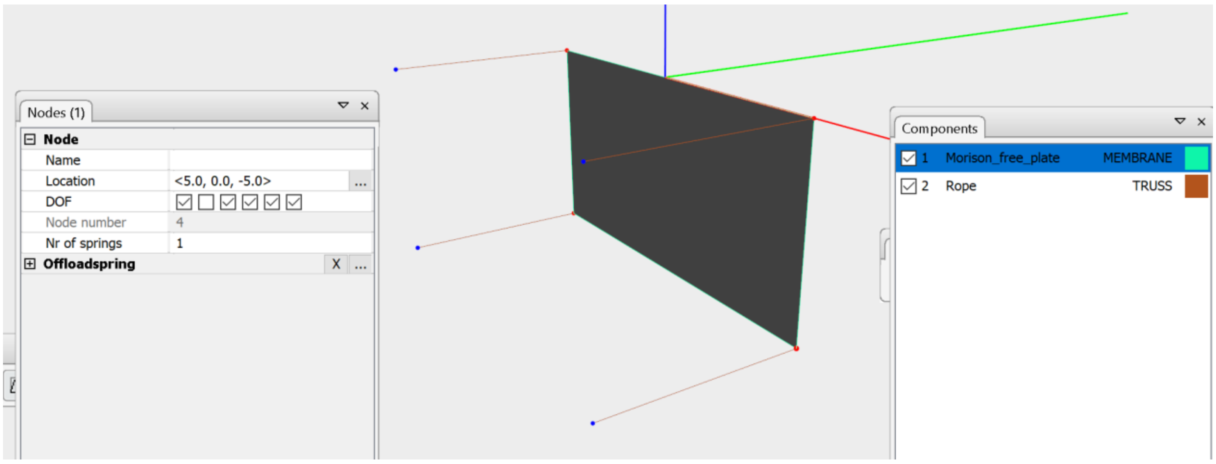


Figure 1 Analysis model with one shell element, and 4 truss elements

The analysis model consists of one shell element with trusses at each corner to easily read resulting forces. The model is suppressed from motions other than in the *y*- direction (normal to the plane of the plate). The structural data for the case study is given in Table 1.

Table 1 Structural data for case study 1

Height of plate [m]	10
Width of plate [m]	5
Area of plate [m2]	50
Current velocity [m/s]	1

Figure 2 shows a comparison of axial forces in each of the truss elements between AquaSim and analytical formulae. The analytic formula is simply applying Equation 1, inserting values for this case. Forces has been calculated for a variation of drag coefficients where the current velocity is constant, and results are given in Figure 2.

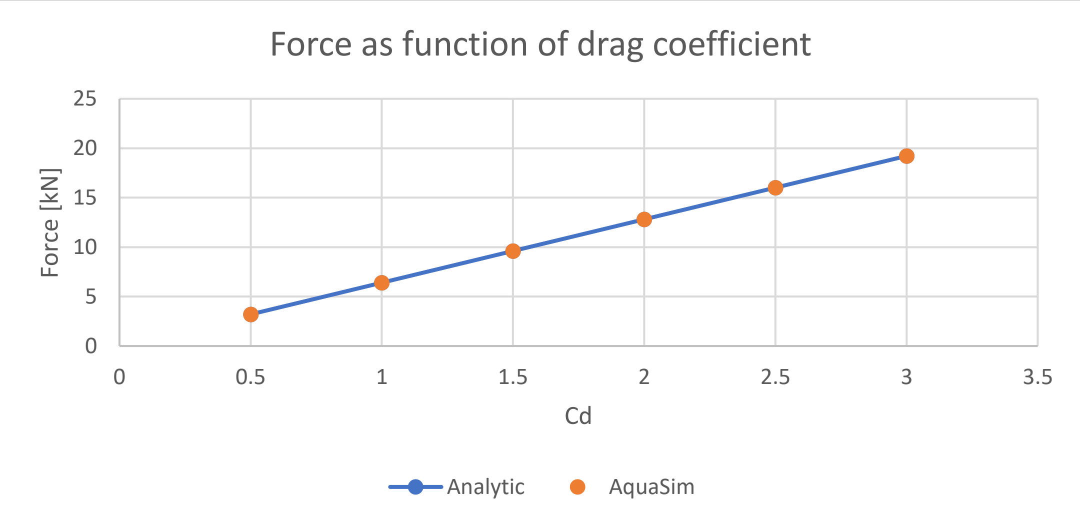


Figure 2 Force as a function of drag coefficient

3.1 Dynamic response of one element plate

This chapter presents case study of dynamic response of the same plate of one element. Figure 3 shows the model in the dynamic case where an offloaded spring has been introduced to the plate corners. A pretension of 0.1 has been introduced to the truss elements meaning the non-strained length of them is 4.5 meters, instead of the modelled length of 5 meters.

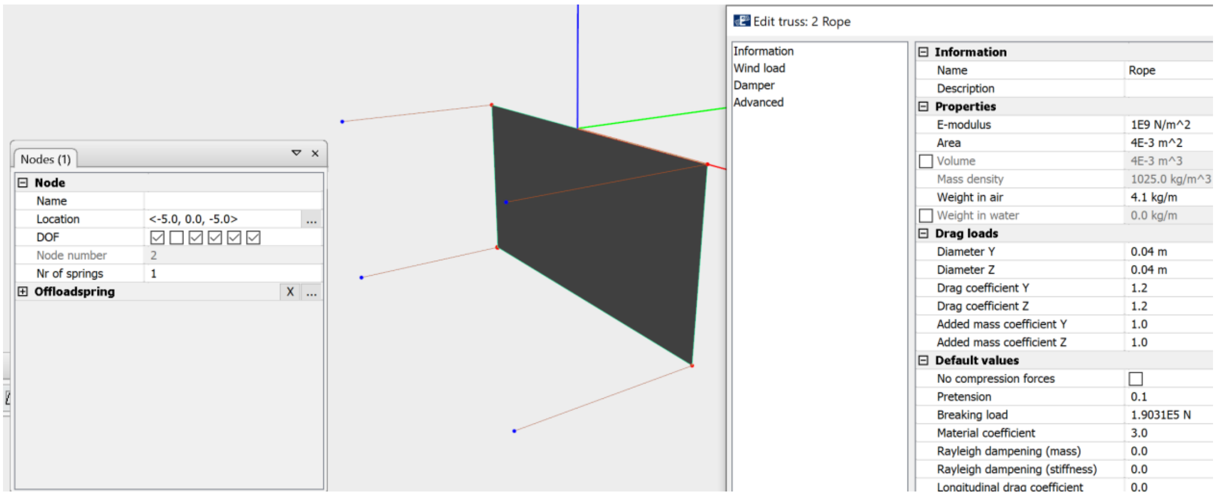


Figure 3 Offloaded spring activated together with a pretension in the trusses. The pretension is 0.1, meaning the length of the truss without pretension is 4.5 meters

Structural data for the model is provided in Table 2.

Table 2 Structural data for case study of dynamic response

Length truss, nominal [m]	5
Pre-strain coefficient	0.1
Tension free rope length [m]	4.5
Elastic modulus, rope [N/m2]	1.00E+09
Cross sectional area rope [m2]	4.00E-03
Density plate [kg/m2]	1025
Density water [kg/m2]	1025
Thickness plate [m]	1.00E-02

In the AquaSim model, an offloaded spring is taken off when the dynamic time domain analysis starts up. Figure 4 shows a timeseries for response in AquaView.

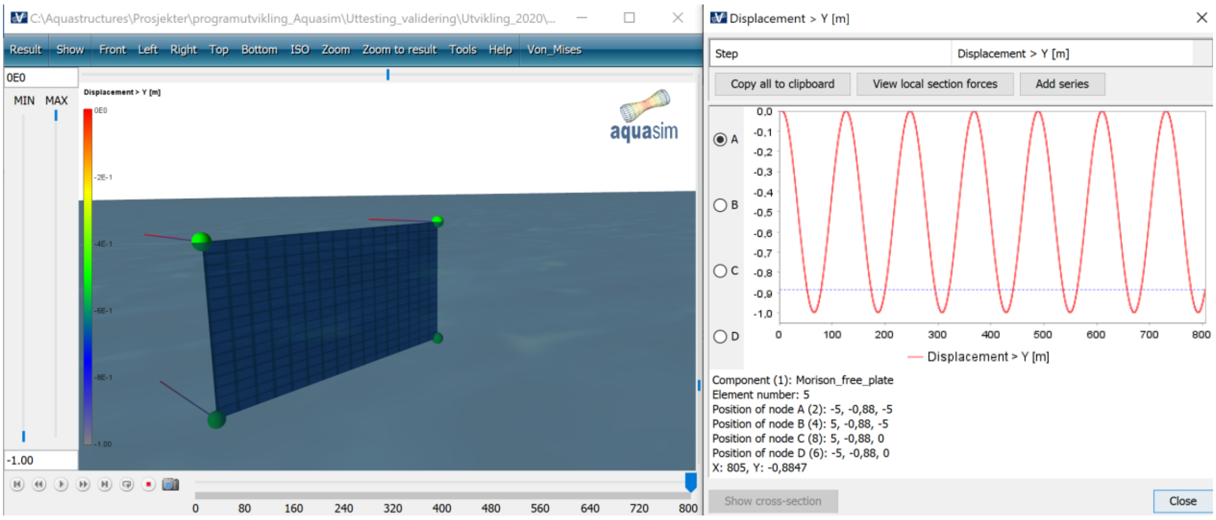


Figure 4 Displacement as a function of axial load

As seen from Figure 4, the response is sinusoidal with an amplitude of 0.5 m and a mean value of -0.5 m. This is as expected since the amplitude is 0.5 m when the offloaded spring is taken off. The response curve has been used to find the eigenperiod of the system from the AquaSim calculation. The derived eigenperiod has been compared to analytical results and results are shown in Figure 5.

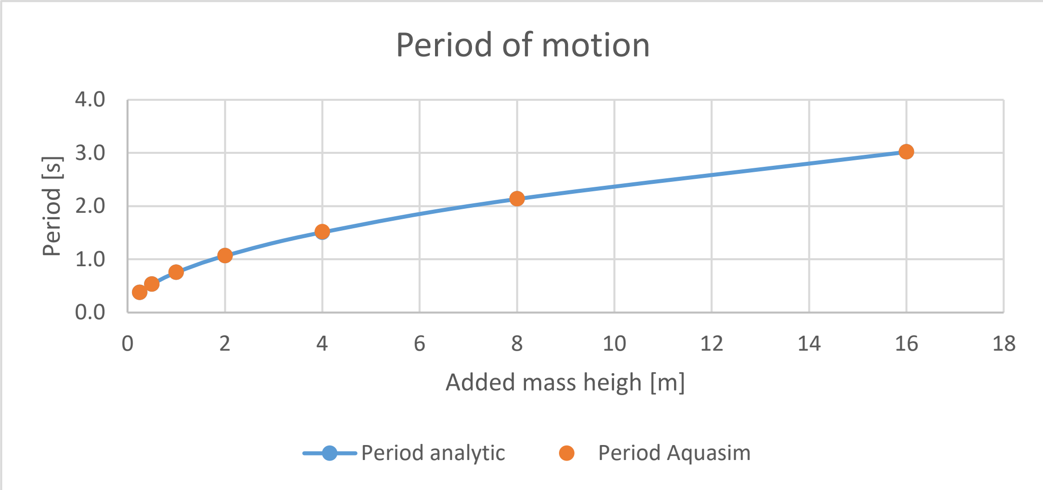


Figure 5 Period of the response motion as a function of added mass. AquaSim results compared to analytical formulae

As seen from Figure 5, the results match exactly as expected.

4 Case study 2, Tarp and shell exposed to current velocity

A tarp has been modelled with several elements in AquaSim, as seen from Figure 6.

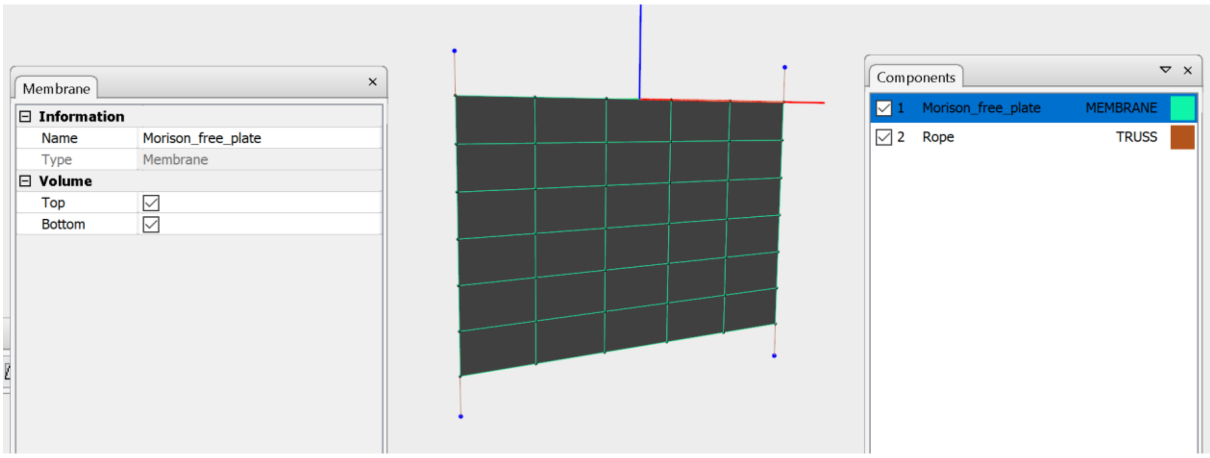


Figure 6 Tarp model

Table 3 shows the main data for the tarp/plate-model.

Table 3 Structural data for case study 2

Height of plate [m]	20
Width of plate [m]	30
Area of plate [m2]	600
Current velocity [m/s]	1.0
Drag coefficient, cd	1.0

Figure 7 shows the tarp exposed to lateral loads from pressure applying the drag loads cross flow for the Morison free plate formulation.

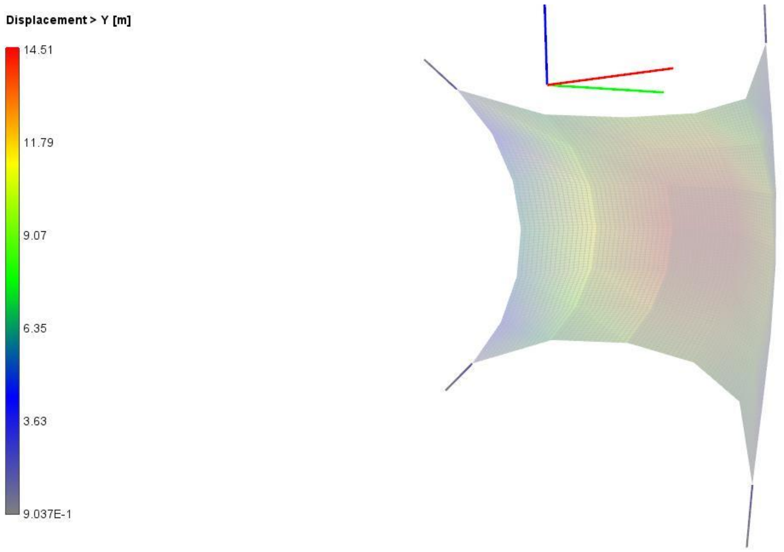


Figure 7 Flexible tarp with Morison free plate load formulation

The load formulation for the AquaSim model is then switched to shell elements with bending stiffness. Two cases of different bending stiffness are applied, called "Soft shell" and "Stiff shell". Comparison of results between tarp, shell element with bending stiffness and analytical calculations are presented in Figure 8. Analytical calculations are based on Equation 1.

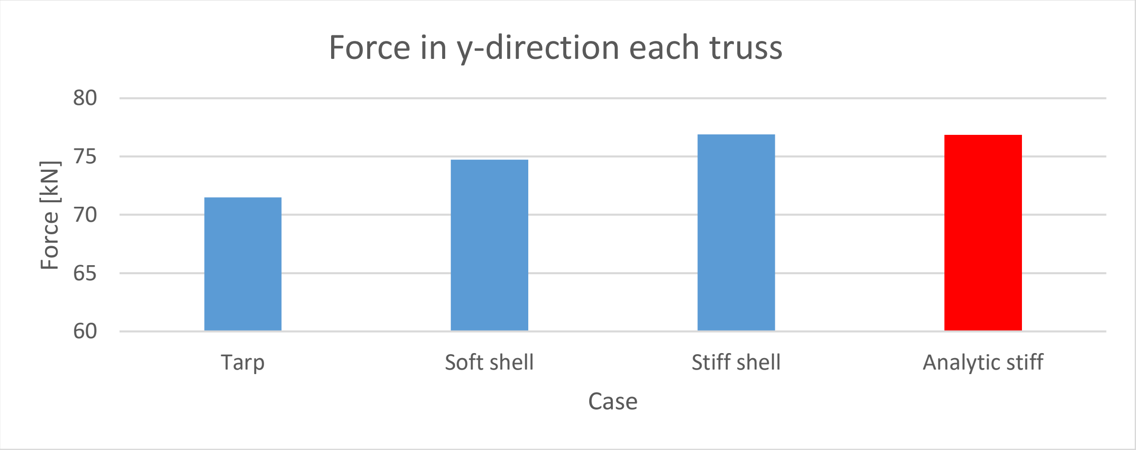


Figure 8 y-component of axial load in each truss at the corner. AquaSim results in blue and analytical results in red

As seen from Figure 8, the stiffer the plate is the closer results are to analytic results assuming a fully stiff plate. The softer the plate, the lower the forces. This is plausible since the cross flow area to the plate is getting smaller, the more the plate deforms. Figure 9 shown the displacement of the stiff plate. This shown why the response is the same for the stiff shell model as for the analytical results based on stiff plate.

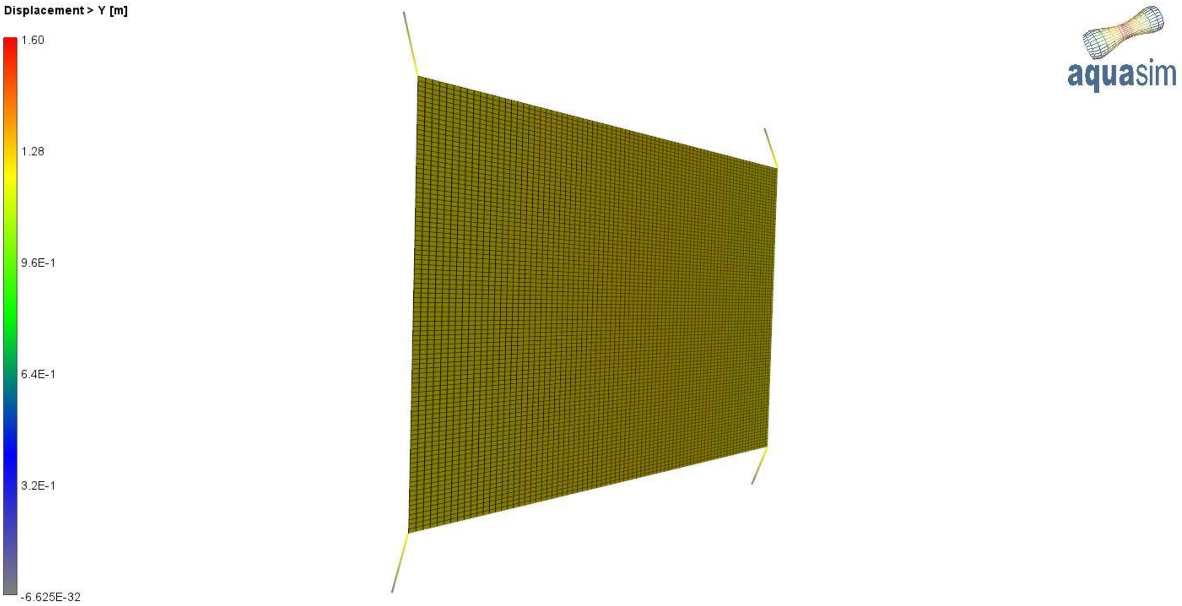


Figure 9 Displacement pattern of stiff shell

5 Conclusions

Based on the analysis carried out in this document it is concluded that the Morison free plate load formulation is a load formulation working well and that it can be useful.

6 References

1. Aquastructures (2020) "The AquaSim Package user manual" Aquastructures report TR-30000-2049-1
2. Aquastructures (2012) "Verification and benchmarking of AquaSim, a softwaretool for safety simulation of flexible offshore facilities exposed to environmental and operational loads", Aquastructures report 2012-1755-1