Cresting Wave Factor

Nonlinear wave kinematics in AquaSim

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Summary:

This report summarizes implementation and validation of the functionality Cresting Wave Factor in AquaSim. Numerical analysis has been compared to analytical calculations.

Shell and beam elements have been used in this study. Comparison of numerical analysis and analytic calculations show good correspondence.

1	20.09.2021	ISH	AJB	Cresting Wave Factor
2	22.09.2022	ISH	AJB	Corrections
3	07.08.2022	ISH	AJB	Corrections in ch. 2.2
Revision no.	Date	Author	Verified by	Description

Content

1 Introduction

The functionality Cresting Wave Factor have been introduced to AquaSim. This is a factor to account for nonlinear wave kinematics due to breaking waves.

The classical linear sinusoidal wave theory (Airy waves) assumes waves of low steepness. Meaning that the crest height is much lower than the wavelength. A breaking wave is a wave whose steepness reaches a critical level of 1/7. These types of waves contain significantly more energy than linear waves of lower steepness. Wave crests break when the horizontal component of the fluid particle velocity is equal, or greater to, the wave propagation velocity. This effect may be of significance for structures situated in the water line, where the increased fluid particle velocity leads to higher forces on the structure.

The implementation- and validation of the Cresting Wave Factor in AquaSim is described in the succeeding chapters.

1.1 Wave steepness

The wave steepness (h_{crest}) is defined as the ration of wave height (H) to the wavelength (λ): $h_{crest} = H/\lambda$, see Figure 1. The maximum steepness of a wave propagating in infinite water depth is $h_{crest} = 1/7 \approx 0.1428$. The wave height is equal to two times the wave amplitude, $H = 2 \cdot \zeta_A$ [m].

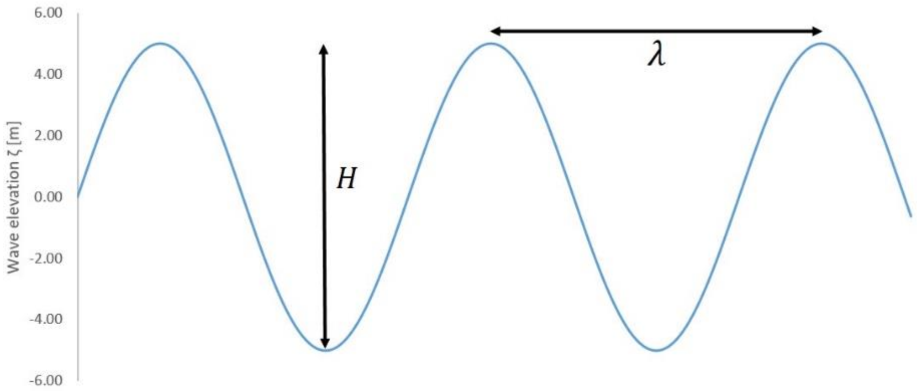

Figure 1 Wave height and wavelength

2 Theoretical formulation

2.1 Linear fluid particle velocity above mean water line (MWL)

In AquaSim, the horizontal component of the fluid particle velocity due to waves is calculated from linear wave theory (see e.g. (Faltinsen, 1990) pp. 16):

$$u = \omega \zeta_A e^{kz} \sin(\omega t - kx)$$

Equation 1

where

- u is the linear fluid velocity due to waves,
- ω is wave frequency,
- ζ_A is wave amplitude,
- $k = \omega^2/g$ is wave number (g is gravitational acceleration),
- z is vertical position of fluid particle,
- t is time variable,
- x is position along x-axis.

The fluid particle velocity change as a function of vertical position, this is seen as the kz-part of Equation 1. In AquaSim, the velocity above the MWL is equal to the velocity at MWL. The consequence of this is that the kz-part will never have positive values.

2.2 Nonlinear fluid particle velocity above mean water line (MWL)

Two equations for the nonlinear horizontal component of fluid particle velocity above MWL are suggested:

$$u_{NL}(z) = u(z)\sqrt{\frac{(\kappa^2 - 1)z}{h_{crest}} + 1}$$

Equation 2

$$u_{NL}(z) = u(z) \cdot \kappa^{\left(\frac{z}{h_{crest}}\right)}$$

Equation 3

where

- u_{NL} is the nonlinear horizontal component of the fluid particle velocity,
- u is the linear horizontal component of the fluid particle velocity,
- κ is the ratio between velocity on top of wave crest and linear crest velocity,
- z is position above MWL,
- $h_{crest} = H/\lambda = 2\zeta_A/\lambda$ is the wave steepness.

In AquaSim, the nonlinear fluid particle velocity is set to have linear increase upwards above the mean water line, which give a distribution between Equation 2 and Equation 3:

$$u_{NL}(z) = u(z) + (\kappa - u(z)) \cdot \left(\frac{z}{h_{crest}}\right)$$

Equation 4

A representation of Equation 2, Equation 3 and Equation 4 is shown in Figure 2, where κ is plotted as a function of wave steepness.

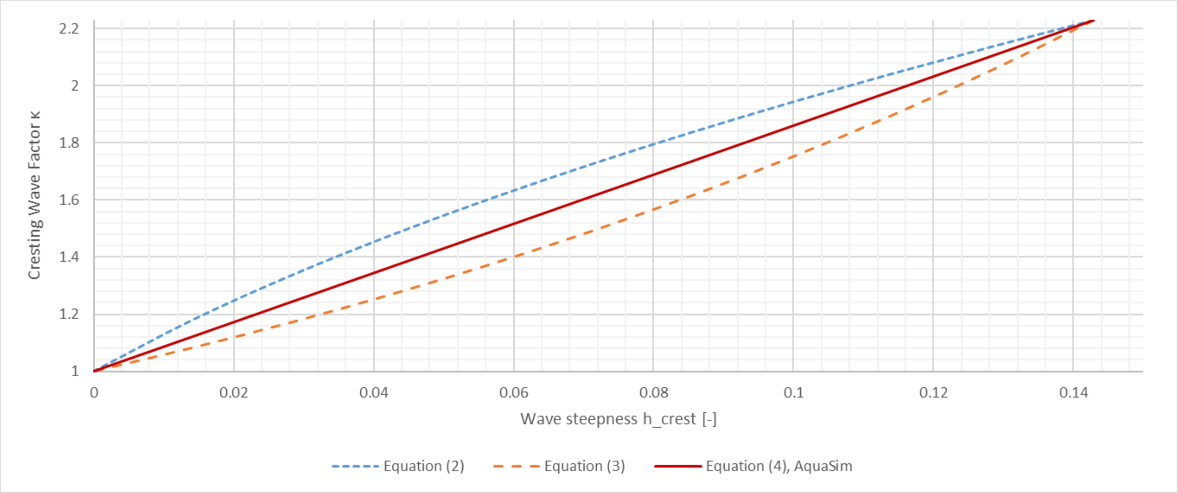

Figure 2 Cresting Wave Factor

In AquaSim, Equation 4 is implemented and κ is the input value (Cresting wave factor). Between the mean water line and top of the crest, AquaSim will interpolate κ. This is illustrated in the figure below.

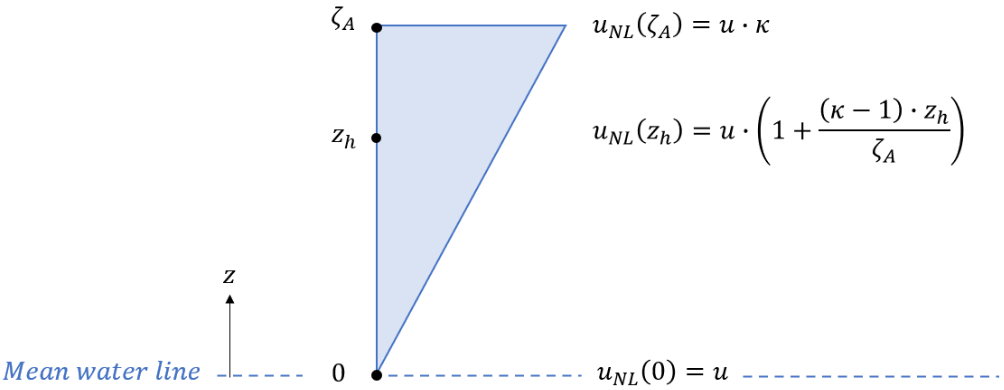

Figure 3 Interpolation of κ in AquaSim

The default value of Cresting Wave Factor is 2.23 (which is the intersecting point between Equation 2 and Equation 3 in Figure 2 with a steepness of 1/7). Note that the Cresting Wave Factor is equal to 1 below the mean water line (MWL).

3 Validation

3.1 Case A1 (Shell and Morison free plate)

In this case study, the Cresting Wave Factor is applied to a membrane element. The membrane type is *Shell* in combination with load formulation *Morison free plate*. Consider a membrane element restrained by four trusses, as depicted in Figure 4. Sinusoidal waves are applied normal to the shell element. No current is present.

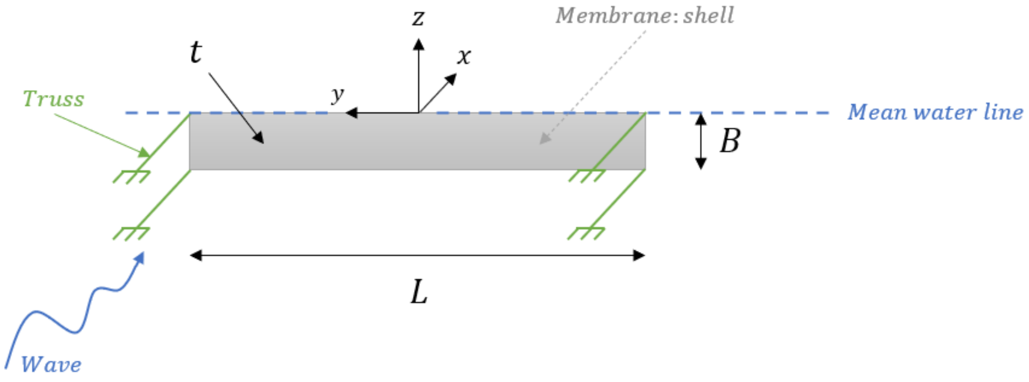

Figure 4

The validation is carried out by consider the force in the upper truss element (see green arrow in figure above). The Cresting Wave Factor is validated for three positions above MWL, as illustrated in Figure 5.

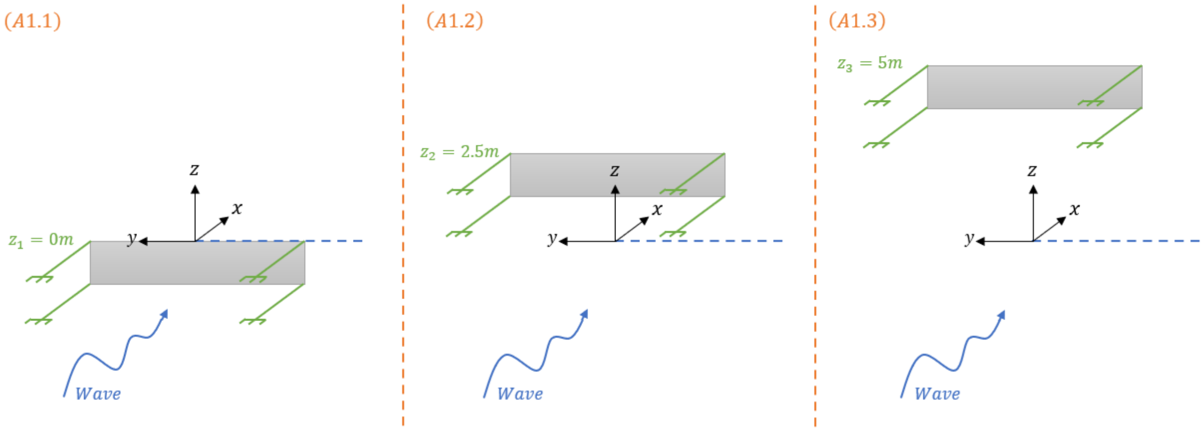

Figure 5

The generalized Morison equation for *Morison free plate* in normal direction is (see (Aquastructures AS, 2021g)):

$$F_N = \underbrace{\frac{1}{2}\rho C_D A \cdot (u_N - \dot{\eta}_N)|u_N - \dot{\eta}_N|}_{\text{Drag force}} \quad + \quad \underbrace{\rho V a_N}_{\text{Froude-Kriloff \& diffraction force}} \quad + \quad \underbrace{\rho A C_a (a_N - \ddot{\eta}_N)}_{\text{Added mass \& damping force}} \quad [N]$$

Equation 5

where

- F_N is the force due to waves in normal direction,
- ρ is fluid density,
- C_D is drag coefficient,
- $A = L \cdot B$ is the area of shell exposed to fluid,
- $u_N = u_{N(C)} + u_{NL(w)}$ is fluid particle velocity due to current and waves ($u_{N(C)}$ is current velocity and $u_{NL(w)}$ is the nonlinear fluid particle velocity due to waves),
- $\dot{\eta}_N$ is velocity of shell in normal direction,
- $V = A \cdot t$ is the volume of shell exposed to fluid,
- a_N is the fluid particle acceleration due to waves in normal direction,
- C_a is the added mass coefficient. Note, this coefficient has the unit meter [m],
- $\ddot{\eta}_N$ is the acceleration of the shell in normal direction.

The following assumptions and simplifications are applied:

- Static: the shell is assumed static. That is, the shell has no forward speed or acceleration. This leads to $\dot{\eta}_N = \ddot{\eta}_N = 0$,
- No current: no current is present, $u_{N(C)} = 0$. Hence, $u_N = u_{NL(w)}$, where $u_{NL(w)}$ is found from Equation 4,
- Fluid acceleration: the fluid particle acceleration in normal direction corresponds to the horizontal component of fluid acceleration, $a_N = a_1$. a_1 is calculated according to (Faltinsen, 1990) pp. 16:

$$a_1 = \omega^2 \zeta_A e^{kz} cos(\omega t - kx) \quad [m/s2]$$

By inserting above simplifications and assumptions into Equation 5 we get:

$$F_N = \quad \frac{1}{2} \rho C_D A \cdot u_{1NL(w)} \left| u_{1NL(w)} \right| \quad + \quad \rho V a_1 \; + \; \rho A C_a a_1 \quad [N]$$

Equation 6

The force in the truss becomes:

$$F_{truss} = \frac{F_N}{\#_{truss}} \quad [N]$$

Equation 7

where F_N is according to Equation 6 and $\#_{truss}$ is the number of trusses (which is 4). Technical specifications for the case study are provided in Table 1.

Table 1 Technical specifications

Technical data	**Abbreviation**	**Value**
Length, shell	L	10 m
Width, shell	B	0.1 m
Area, shell	A	1m
Thickness, shell	t	0.1 m
Fluid density	ρ	1025 kg/m3
Gravitational acceleration	g	9.81 m/s2
Drag coefficient	C_D	1 -

Added mass coefficient	C_a	0 m
Wave amplitude	ζ_A	5 m
Wave steepness	$h_{crest} = 2\zeta_A/\lambda$	1/7
Wave length	$\lambda = \frac{1}{h_{crest}} 2\zeta_A$	70 m
Wave period	$T = \sqrt{\frac{1}{h_{crest}} 2\zeta_A 2\pi \frac{1}{g}}$	6.6958 s
Wave frequency	$\omega = 2\pi/T$	0.94 s-1
Wave number	$k = \omega^2/g$	0.09 m
Number of steps per wave	Num	50 -
Time increment	$dt = T/Num$	0.13 s
Number of waves	$t = 3T$	20.08 s
Location shell, along x-axis	x	0 m
Vertical position truss, case A1.1	z_1	0 m
Vertical position truss, case A1.2	z_2	2.5 m *)
Vertical position truss, case A1.3	z_3	5 m *)
Cresting wave factor, input AquaSim	κ	2.23 -
Cresting wave factor, case A1.1	$\kappa(z_0 = 0m)$	1 -
Cresting wave factor, case A1.2	$\kappa(z_{2.5} = 2.5m)$	1.615 -
Cresting wave factor, case A1.3	$\kappa(z_5 = 5m)$	2.23 -

*) Adjustments in kz-part of fluid particle velocity- and acceleration must be made according to chapter 2.1.

Each truss are 4 meters long, with a E-modulus of 1.0E+11N/m2 and area of 0.1m2. Other parameters are equal to zero. This to prevent the truss to contribute to damping and buoyancy. The only function of the truss is to restrain the shell and take up axial force.

3.1.1 Results

Results from analytic calculations and AquaSim analysis is presented in Figure 6-Figure 8.

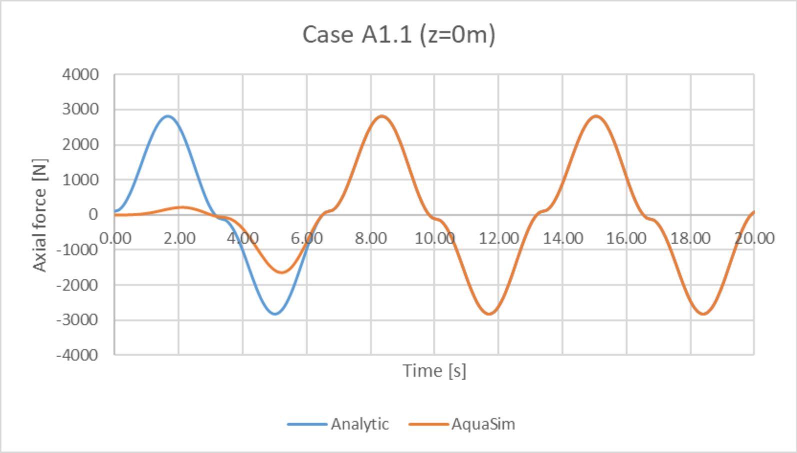

Figure 6 Comparison analytic- and AquaSim results

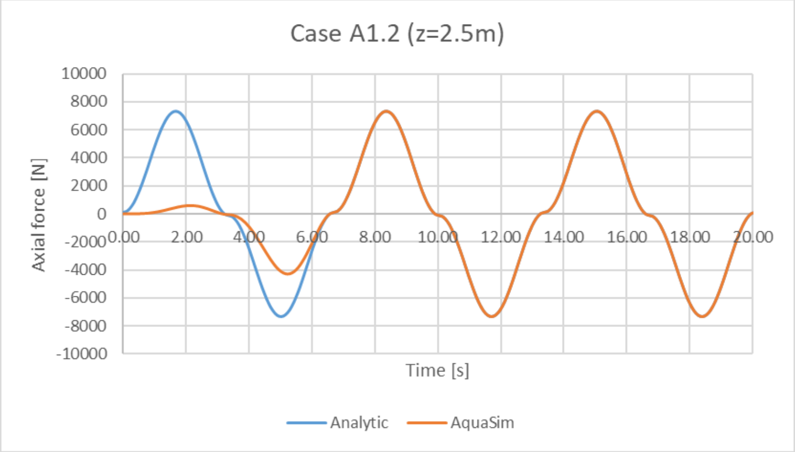

Figure 7 Comparison analytic- and AquaSim results

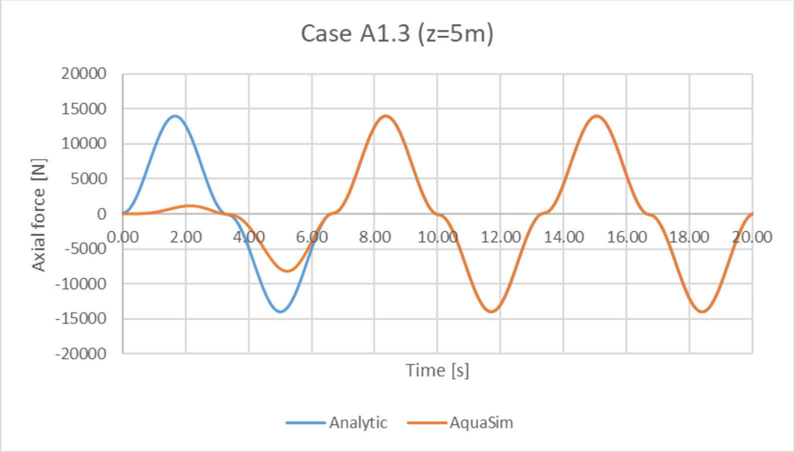

Figure 8 Comparison analytic- and AquaSim results

Within the first wave cycle, AquaSim results deviates from analytical. This is due to AquaSim applying environmental loads by linear increase. Read more about it in the AquaSim Theory Manual, chapter *Properties of time domain simulation*. The deviations are accepted.

In this study, the compressive force in the truss is of the same magnitude as axial tension. When an object moves out of water (due to wave trough passes) the axial force would in reality approach 0 N. In AquaSim, the Water volume correction is set to Normal. Implying that correction for in-and-out-of water is omitted. This is only done for reasons of simplicity.

The results show good correspondence. The larges deviation between extreme values (maximum and minimum) is -0.3% for the investigated cases. The validation is regarded successful.

3.2 Case A2 (Shell and Morison free plate)

This case study is identical with the one presented in chapter 3.1. For illustrations, equations, assumptions and technical specifications reference is made to this chapter. The only difference is that the Cresting Wave Factor is changed from 2.23 to 2.0. This will result in the following corrections from Table 1:

Table 2 Technical specifications

Technical data	Abbreviation	Value
Cresting wave factor, input AquaSim	κ	2.0-
Cresting wave factor, case A2.1	$\kappa(z_1)$	1-
Cresting wave factor, case A2.2	$\kappa(z_2)$	1.5-
Cresting wave factor, case A2.3	$\kappa(z_3)$	2.0-

3.2.1 Results

Results from analytic calculations and AquaSim analysis are presented in Figure 9-Figure 11.

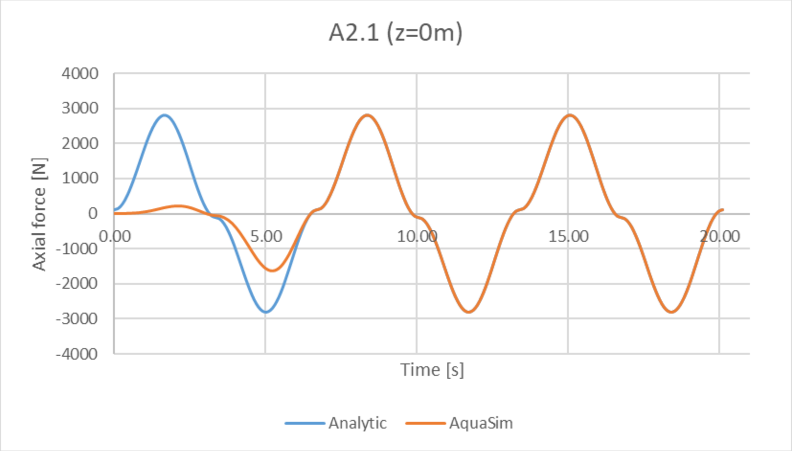

Figure 9 Comparison analytic- and AquaSim results

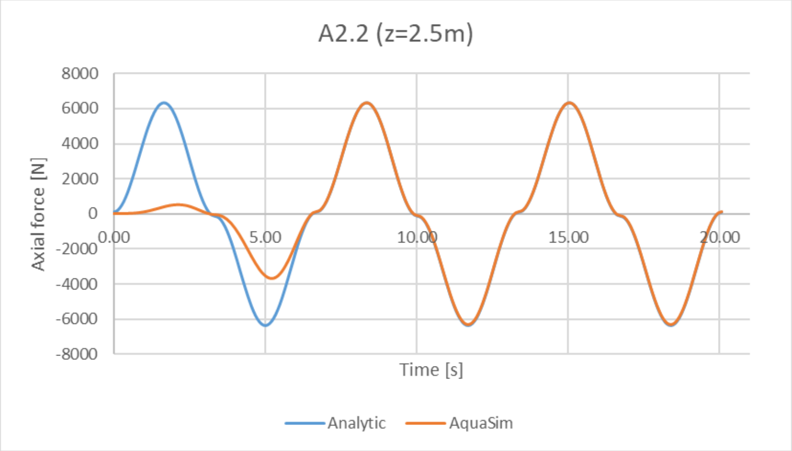

Figure 10 Comparison analytic- and AquaSim results

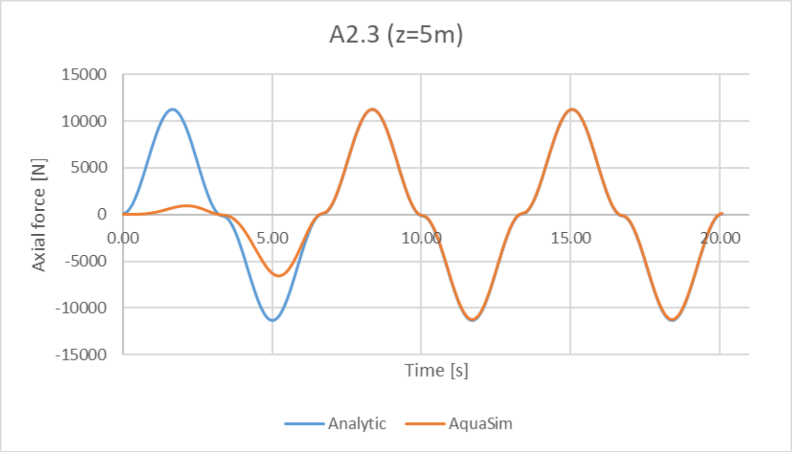

Figure 11 Comparison analytic- and AquaSim results

In this study, the compressive force in the truss is of the same magnitude as axial tension. When an object moves out of water (due to wave trough passes) the axial force would in reality approach 0 N. In AquaSim, the Water volume correction is set to Normal. Implying that correction for in-and-out-of water is omitted. This is only done for reasons of simplicity.

The results show good correspondence. The larges deviation between extreme values (maximum and minimum) is -0.27% for the investigated cases. The validation is regarded successful.

3.3 Case B1 (Beam and Morison submerged)

In this case study, the Cresting Wave Factor is applied to a beam element. The load formulation is Morison submerged. Consider a beam restrained with two trusses, as depicted in Figure 12. The beam is exposed to sinusoidal waves in x-direction, no current.

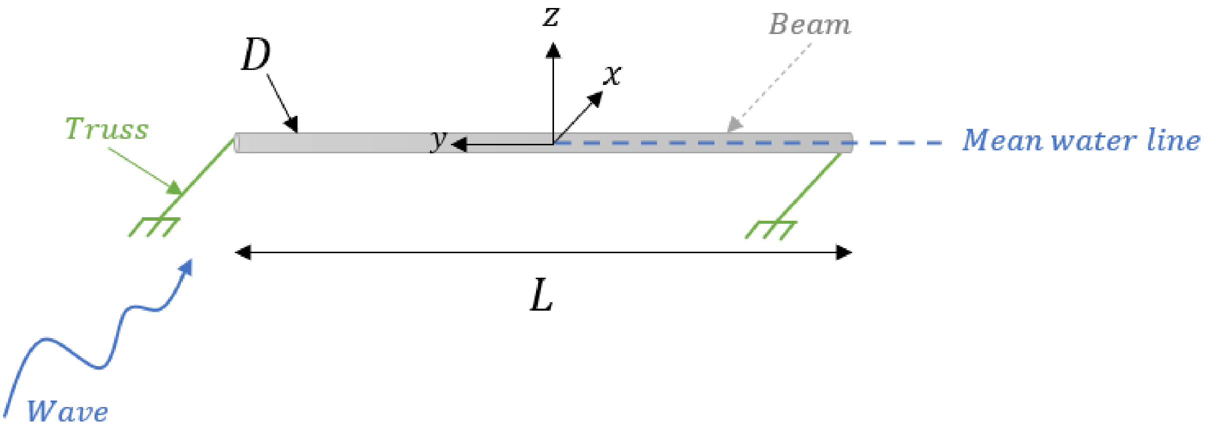

Figure 12

The validation is carried out by consider the for in one truss (see green arrow in figure above). The Cresting Wave Factor is validated for three positions above MWL, as illustrated in Figure 13.

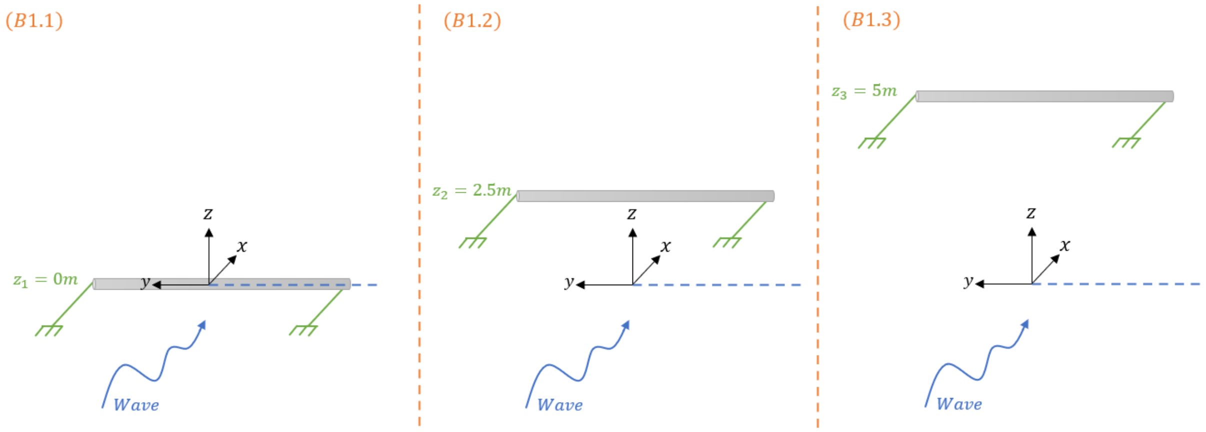

Figure 13

The generalized Morison equation for Morison submerged is (Faltinsen, 1990) pp. 225:

$$F_i = \underbrace{\frac{1}{2}\rho C_D A \cdot (u_i - \dot{\eta}_i)|u_i - \dot{\eta}_i|}_{\text{Drag force}} + \underbrace{\rho V C_M a_i}_{\text{Froude-Kriloff \& diffraction force}} - \underbrace{\rho A (C_M - 1)\ddot{\eta}_i}_{\text{Added mass \& damping force}} \quad [N]$$

Equation 8

where

- F_i is the force due to waves in $i = [1, 2, 3] = [x, y, z]$ direction,
- ρ is fluid density,
- C_D is drag coefficient,

- $A = L \cdot D$ is the area of the beam exposed to fluid,
- $u_i = u_{i(C)} + u_{i(w)}$ is fluid particle velocity due to current ($u_{i(C)}$) and waves($u_{i(w)}$),
- $\dot{\eta}_i$ is the velocity of the beam,
- $V = (\pi D^2/4) \cdot L$ is the volume of the beam exposed to fluid,
- $C_M = 1 + C_a$ is the mass coefficient,
- a_i is the fluid particle velocity due to waves,
- $\ddot{\eta}_i$ is the acceleration of the beam.

The following assumptions and simplifications are applied:

- Static: the beam is assumed static. That is, the beam has no forward speed or acceleration. This leads to $\dot{\eta}_i = \ddot{\eta}_i = 0$,
- No current: the beam is not exposed to current, $u_{i(C)} = 0$. Hence, $u_i = u_{i(w)}$,
- The nonlinear horizontal component of fluid particle velocity $u_{1NL(w)}$ is found according to Equation 4,
- Vertical component of fluid particle velocity is calculated according to (Faltinsen, 1990) pp. 16:
$$u_{3(w)} = \omega \zeta_A e^{kz} \cdot cos(\omega t - kx) \quad [m/s]$$
- Acceleration: the horizontal fluid particle acceleration due to waves is calculated according to (Faltinsen, 1990) pp. 16:
$$a_1 = \omega^2 \zeta_A e^{kz} cos(\omega t - kx) \quad [m/s2]$$

By inserting the abovementioned assumptions and simplifications into Equation 8 we get the following expression for the force on the beam in x-direction (i.e. $i = 1$):

$$F_1 = \frac{1}{2}\rho C_D A \cdot u_{1NL(w)}\sqrt{u_{1NL(w)}^2 + u_{3(w)}^2} \quad + \quad \rho V(1 + C_a)a_1 \quad [N]$$

Equation 9

The force in the truss becomes:

$$F_{truss} = \frac{F_1}{\#_{truss}} \quad [N]$$

Equation 10

where F_x is calculated according to Equation 9 and $\#_{truss}$ is the number of truss (which is 2). Technical specifications for the case study are provided in Table 3.

Table 3 Technical specifications

Technical data	**Abbreviation**	**Value**
Length, beam	L	10 m
Diameter, beam	D	0.1 m
Volume, beam	V	0.08 m3
Fluid density	ρ	1025 kg/m3
Gravitational acceleration	g	9.81 m/s2
Drag coefficient	C_D	1 -
Mass coefficient	C_M	0 m
Wave amplitude	ζ_A	5 m

Wave steepness	$h_{crest} = 2\zeta_A/\lambda$	1/7
Wave length	$\lambda = \frac{1}{h_{crest}} 2\zeta_A$	70 m
Wave period	$T = \sqrt{\frac{1}{h_{crest}} 2\zeta_A 2\pi \frac{1}{g}}$	6.6958 s
Wave frequency	$\omega = 2\pi/T$	0.94 s-1
Wave number	$k = \omega^2/g$	0.09 m
Number of steps per wave	Num	50 -
Time increment	$dt = T/Num$	0.13 s
Number of waves	$t = 3T$	20.08 s
Location shell, along x-axis	x	0 m
Vertical position truss, case B1.1	z_1	0 m
Vertical position truss, case B1.2	z_2	2.5 m *)
Vertical position truss, case B1.3	z_3	5 m *)
Cresting wave factor, input AquaSim	κ	2.23 -
Cresting wave factor, case B1.1	$\kappa(z_0 = 0m)$	1 -
Cresting wave factor, case B1.2	$\kappa(z_{2.5} = 2.5m)$	1.615 -
Cresting wave factor, case B1.3	$\kappa(z_5 = 5m)$	2.23 -

*) Adjustments in kz-part of fluid particle velocity- and acceleration must be made according to chapter 2.1.

Each truss are 4 meters long, with a E-modulus of 1.0E+11N/m2 and area of 0.1m2. Other parameters are equal to zero. This to prevent the truss to contribute to damping and buoyancy. The only function of the truss is to restrain the shell and take up axial force.

3.3.1 Results

Results from analytic calculations and AquaSim analysis are presented in Figure 14-Figure 16.

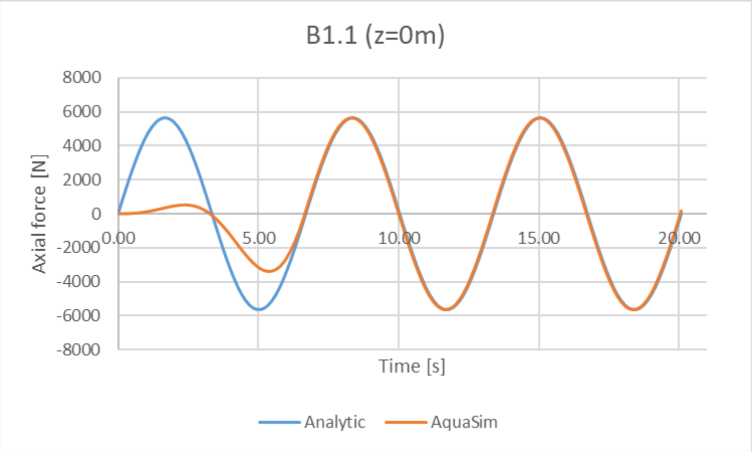

Figure 14 Comparison analytic- and AquaSim results

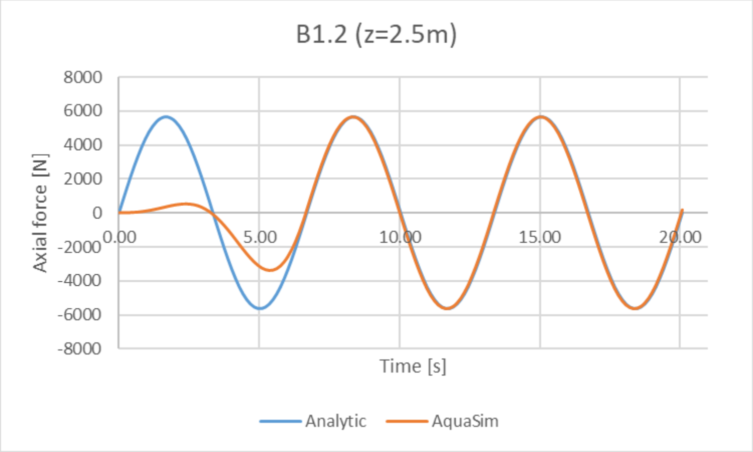

Figure 15 Comparison analytic- and AquaSim results

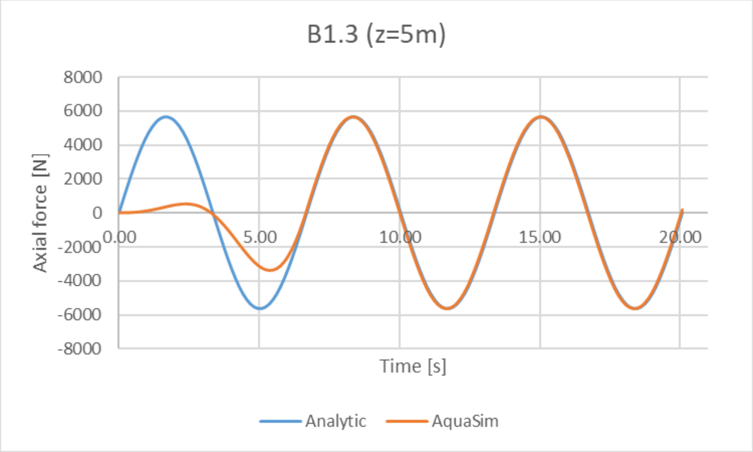

Figure 16 Comparison analytic- and AquaSim results

The results show good correspondence. The larges deviation between extreme values (maximum and minimum) is 0.40% for the investigated cases. The validation is regarded successful.

4 Conclusions

Based on the analysis in this document, it is concluded that the Cresting Wave Factor is working according to expectations and is implemented in AquaSim.

5 References

Aquastructures AS. (2021g). *Morison free plate load formulation to shell and membrane elements.* Techn. rep. TR-FOU-2328-8.

Faltinsen, O. (1990). *Sea Loads on Ships and Offshore Structures.* Cambridge University Press. ISBN 0-521-37285-2.