REPORT

WATER ENTRY AND EXIT

REPORT NO. TR-FOU-1984-3

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AQUASTRUCTURES

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Summary:

This document describes how water entry and exit is handled in AquaSim in the case where user specifies this to be included in the analysis.

The AquaSim approach is verified by comparison to hand calculation.

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1. INTRODUCTION

Running Aquasim default, water entry and exit is not accounted for. There are several ways an AquaSim user can account for water entry and exit. The rule of thumb is that the more refined water entry and exit is accounted for, the slower the convergence properties of the analysis are and the more input need to be provided. This document describes how water entry and exit is handled in AquaSim in the case where user specifies this to be included in the analysis.

2. WATER ENTRY

Water entry is defined as the period where an object, structural member in the analysis enters the water from dry condition. This is shown in Figure 1.

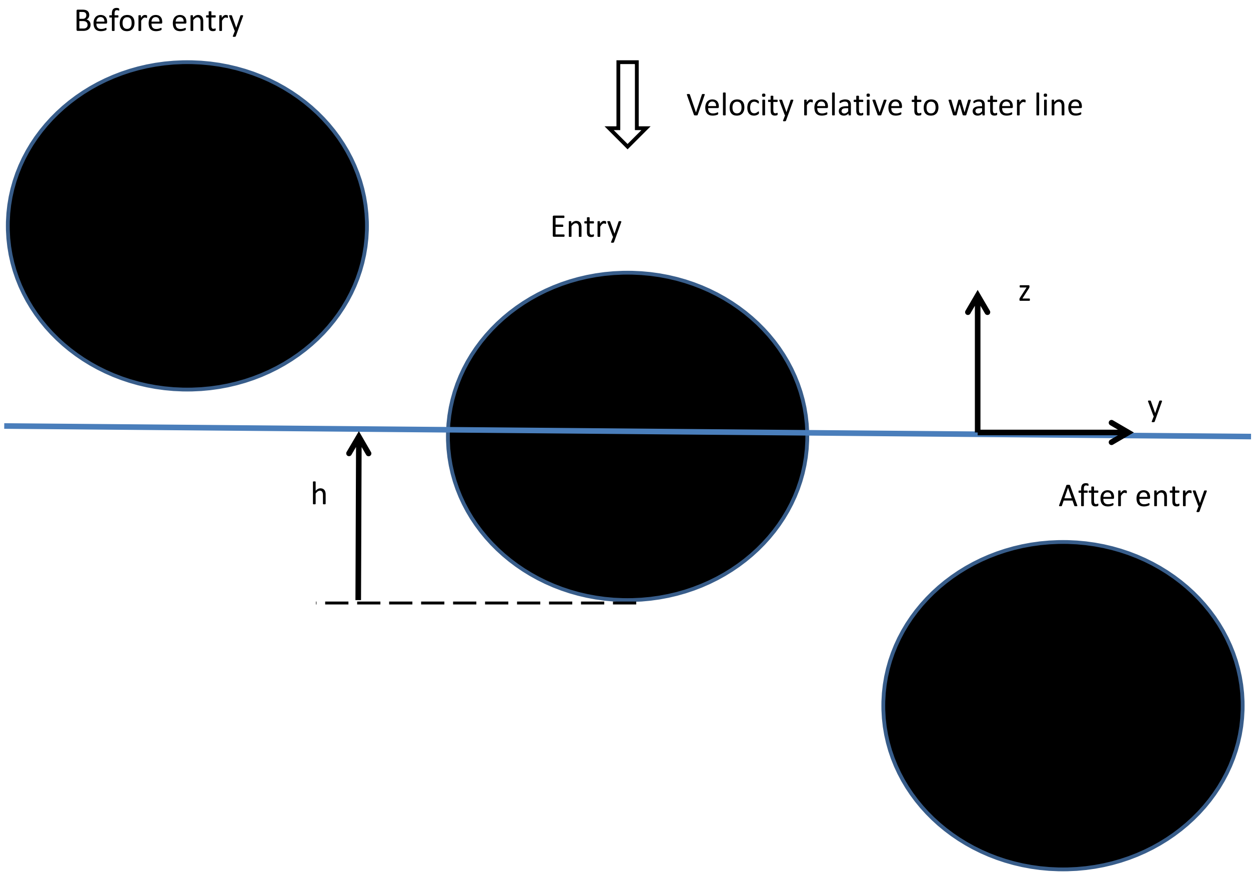

Figure 1 Water entry of circular cylinder. A cross section is seen.

2.1. Vertical motion

Define an orthonormal coordinate system where the x- axis in along the cylinder in the horizontal plane, the z- axis is upwards with origin at the mean water line as shown in Figure 1. In the case where only vertical forces are considered, the vertical force, F_3 acting on the body can be described as (see e.g. Faltinsen 1990, pp 306):

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) - \frac{d}{dt}(A_{33}(\frac{d\eta_3}{dt} - w))$$

Equation 1

Where:

w is the vertical velocity of the water at the considered time instant, $w = \omega\zeta_a \sin(\omega t - kx)$ for the case of a sinusoidal wave traveling along the positive *x*- axis.

ζ_a is the wave amplitude and ω is the wave frequency.

k is the wave number.

η_3 is the vertical location of the object penetrating the water.

ρ is the density of the water and

g is the acceleration constant.

$\Omega(t)$ is the displaced volume at the given time instant

A_{33} is the added mass in the vertical direction.

Equation 1 can be reformulated to

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) - \frac{dA_{33}}{dt}(\frac{d\eta_3}{dt} - w) - A_{33}\frac{d}{dt}(\frac{d\eta_3}{dt} - w))$$

Equation 2

Equation 2 can be expressed as

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) - \frac{dA_{33}}{dt}(\frac{d\eta_3}{dt} - w) - A_{33}(t)\frac{d^2\eta_3}{dt^2} + A_{33}(t)\frac{dw}{dt}$$

Equation 3

Insert $\frac{dA_{33}}{dt} = \frac{dA_{33}}{dh}\frac{dh}{dt}$ where h is the distance from the bottom of the water entering object to the instant position as shown in Figure 1. This means F_3 can be expressed as:

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) - \frac{dA_{33}}{dh}\frac{dh}{dt}(\frac{d\eta_3}{dt} - w) - A_{33}(t)\frac{d^2\eta_3}{dt^2} + A_{33}(t)\frac{dw}{dt}$$

Equation 4

$\frac{dh}{dt}$ can be expressed as $\frac{dh}{dt} = w - \frac{d\eta_3}{dt}$ meaning

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) + \frac{dA_{33}}{dh}(\frac{d\eta_3}{dt} - w)^2 - A_{33}\frac{d^2\eta_3}{dt^2} + A_{33}\frac{dw}{dt}$$

Equation 5

$\frac{d\eta_3}{dt} - w$ is the relative velocity between the object and the water, denoted v_{rel}, $v_{rel} = w - \frac{d\eta_3}{dt}$.
Using this denotation,

$$F_3 = \rho\Omega(t)\frac{dw}{dt} + \rho g\Omega(t) + \frac{dA_{33}}{dh}(v_{rel})^2 - A_{33}\frac{d^2\eta_3}{dt^2} + A_{33}\frac{dw}{dt}$$

Equation 6

As seen from Equation 6 there are 5 load terms to consider. These are handled in AquaSim as described below:

$\rho\Omega(t)\frac{dw}{dt}$ is the Froude Kriloff force. This term is added not only in the *z*- direction, for also in the horizontal plane.

$\rho g\Omega(t)$ as the hydrostatic force (buoyancy) which by definition acts only along the positive z-axis.

$\frac{dA_{33}}{dh}{v_{rel}}^2$ is the slamming force. How this is threated is described in Section 2.3

$A_{33}\frac{d^2\eta_3}{dt^2}$ is the added mass. Added mass as a function of position in terms of *h* in Figure 1 should be given as input. Alternatively default AquaSim values should be used.

$A_{33}\frac{dw}{dt}$ is the diffraction force.

The terms and the load formulation correspond to the load formulation using Morisons formulae. This is seen by rearranging the terms in Equation 6. Denote $\frac{dA_{33}}{dh} = DL\frac{\rho}{2}Cd_{slam}$ where D can be seen as a reference diameter of the object and L a reference length:

$$F_3 = (\rho\Omega(t) + A_{33})\frac{dw}{dt} + DL\frac{\rho}{2}Cd_{slam}(v_{rel})^2 - A_{33}\frac{d^2\eta_3}{dt^2} + \rho g\Omega(t)$$

Equation 7

A_{33} can be given in terms of an added mass coefficient *Ca* then multiplied with the submerged volume to derive the added mass.

As seen by Equation 6 AquaSim needs input in terms of added mass *Am* or coefficient *Ca* given as a function of h (as shown in Figure 1) to account properly for these load terms in the water entry phase. For a circular section, AquaSim have default values that can be used.

2.2. Generalizing to all degrees of freedom.

Consider having modelled a beam or another object using the strip theory as shown in Figure 2. Each element have a local coordinate system where the local x- coordinate is defined as going from node 1 to node 2 of the element and the local z- axis is given as input.

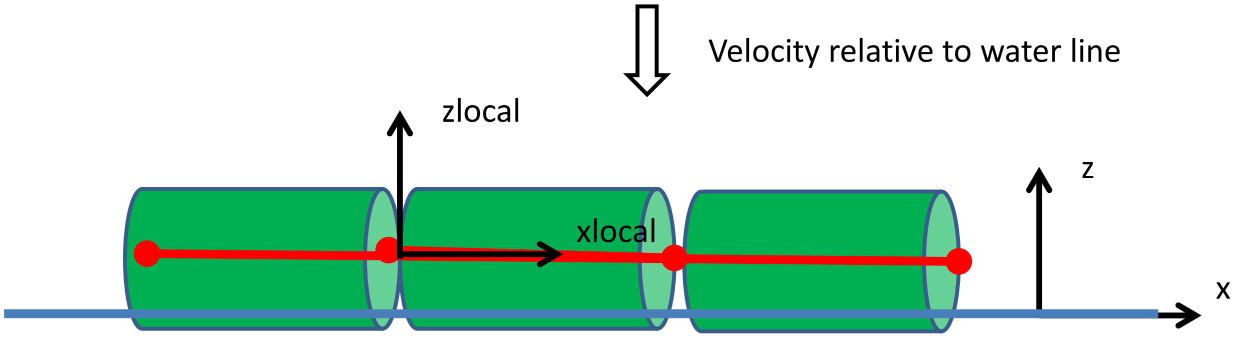

Figure 2 Beam divided to beam elements. Beam has visually been sliced, this can be interpreted as two dimensional strips. i addition a strip is introduced at the middle of the element.

The analysis model exposed to water entry is described by strip theory. Each element is divided to two parts and threated separately. Consider a case as seen in Figure 2 where the local and the global z- axis have the same direction. Let *u,v,w* be the fluid velocity in the *x,y,* and *z*- axis respectively. The terms in Equation 6 are in this case interpreted as:

The Froude Kriloff force in the y- and z- direction is derived as $\rho\Omega(t)\frac{dv}{dt}$ and $\rho\Omega(t)\frac{dw}{dt}$ for the y- and z- axis respectively. $\Omega(t)$ is the submerged volume at the given time instant.

The hydrostatic force, $\rho g\Omega(t)$ does at any case operate directed in the positive z- axis in the global coordinate system.

$\frac{dA_{33}}{dh}v_{rel}{}^{2}$ is the slamming force. How this is threated is described in Section 2.3.

Added mass is added to the mass of the object according to input added mass as a function of displacement of the strip, both horizontally and vertically, A_{22} and A_{33} respectively. Added mass as a function of position in terms of *h* in Figure 1 should be given as input. This goes for both , A_{22} and A_{33}. Alternatively default AquaSim values can be applied.

The diffraction force horizontally and vertically is added to the Froude Kriloff as $A_{22}\frac{du}{dt}$ $A_{33}\frac{dw}{dt}$ respectively where , A_{22} and A_{33} is the added mass at the given submergence.

In the general case, the object may approach the water surface in a skewed manner as shown in Figure 3.

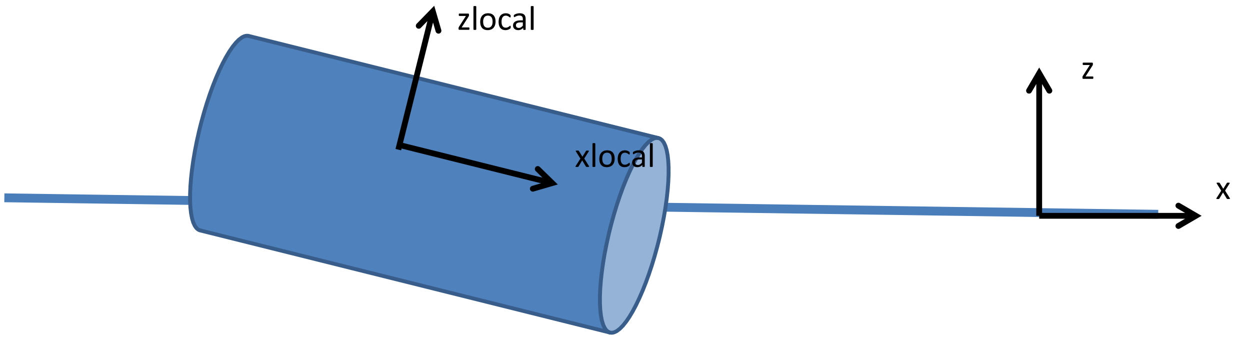

Figure 3 Object entering the water surface in a skewed manner.

In the case seen in Figure 3, numerical integration is used to derive the submerged volume which is used to calculate hydrostatic force and Froude Kriloff for is calculated. Added mass and diffraction is calculated based on the added mass for the actual submergence at each strip with interpolation between the strips.

How the load term proportional to relative velocity squared is threated is shown in Section 2.3.

2.3. Treatment of the "drag" term in the load application formulae

Consider the Morison which in the case of force in the z- (3) direction can be expressed as: (e.g. Faltinsen 1990)

$$F_3 = \frac{\rho_w Cd_z \, Diam_N \, L_0}{2}(u_3 - \dot{r}_3)\sqrt{(u_2 - \dot{r}_2)^2 (u_3 - \dot{r}_3)^2}$$
$$+ \frac{\rho_w Cm_z Diam_z{}^2 L_0 \pi}{4} a_3 - \frac{\rho (Cm_z - 1) Diam_z{}^2 L_0 \pi}{4} \ddot{r}_{3m}$$

Equation 8

for a circular cylinder. In Equation 8, there are only forces in the local y and z direction according to the cross flow principle. $Diam_N$ is a relevant diameter of the object calculated in the direction of the flow based on $Diam_y$ and $Diam_z$ as input. $Cm = Ca + 1$. The load term of Equation 8 proportional to the square of the relative velocity is shown in Equation 9:

$$F_3 = \frac{\rho_w Cd_z \, Diam_N \, L_0}{2}(u_3 - \dot{r}_3)\sqrt{(u_2 - \dot{r}_2)^2 (u_3 - \dot{r}_3)^2}$$

Equation 9

In the case of water entry the cross flow force in the local z- direction is calculated in the same way as in Equation 9 as shown in Equation 10.

$$F_3 = \frac{\rho_w Cd_{slam} \, Diam_N \, L_0}{2}(w - \dot{r}_3)\sqrt{(v - \dot{r}_2)^2 (w - \dot{r}_3)^2}$$

Equation 10

Cd_{slam} can be derived from the input based on the relation $\frac{dA_{33}}{dh} = DL\frac{\rho}{2}Cd_{slam}$. At each strip this value is derived and values are interpolated between the strips. Also forces in the local *y*-direction are calculated. The values for Cd_y in the water entry phase could be given as input, or default AquaSim values may be used. Note that slam calculation is based on having the local *z*-axis pointing in a direction relevant for the slam loads. This should be enabled by the analyst.

2.3.1. AquaSim default values

For a circular cross section AquaSim have default values for parameters relevant for slamming. They are based on values found in Faltinsen (1990) as shown in Figure 4.

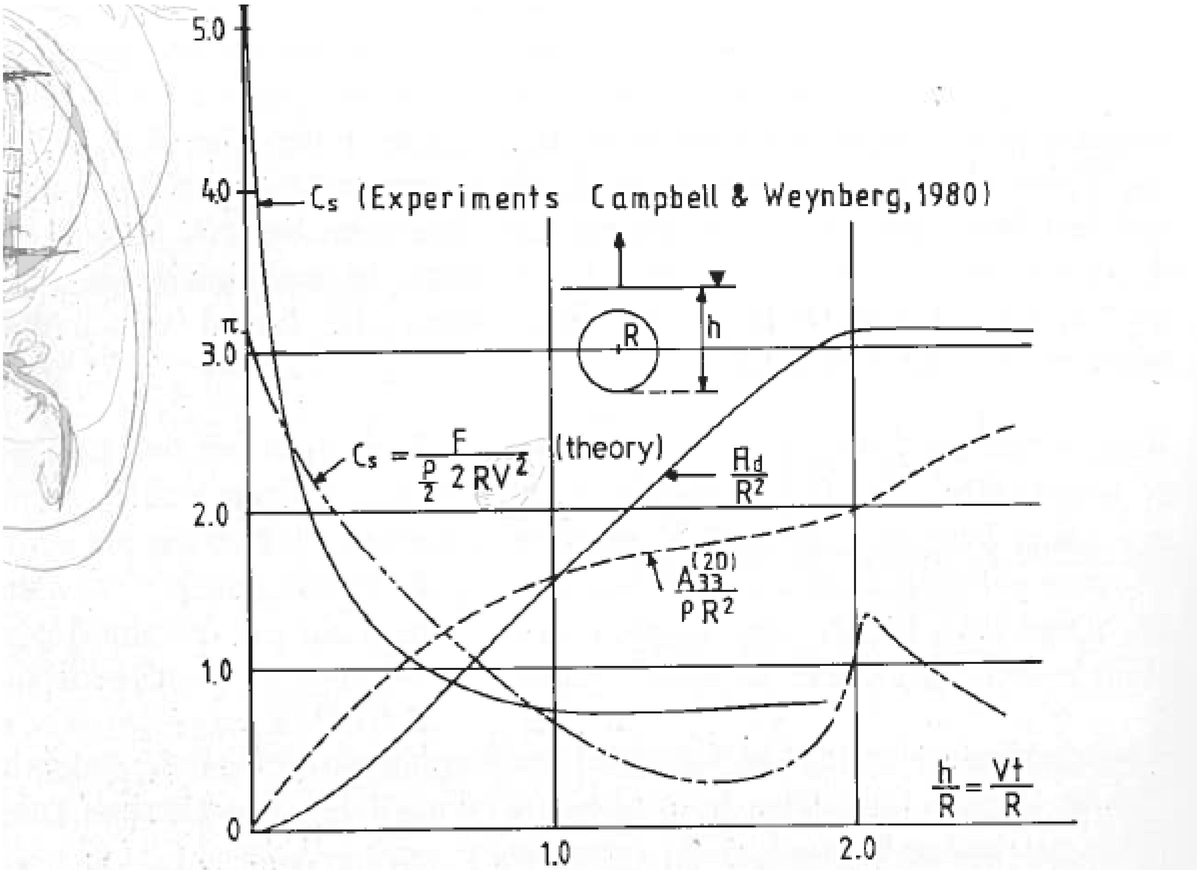

Fig. 9.11. Slamming coefficient C_s, added mass $A_{33}^{(2D)}$ and displaced volume A_d of a circular cylinder as a function of submergence. (F = vertical force, V = constant downward velocity of the cylinder, t = time variable with $t = 0$ corresponding to initial time of impact.)

Figure 4 From Faltinsen (1990)

The AquaSim default values are compared to the two cases in Figure 4 is shown in Figure 5.

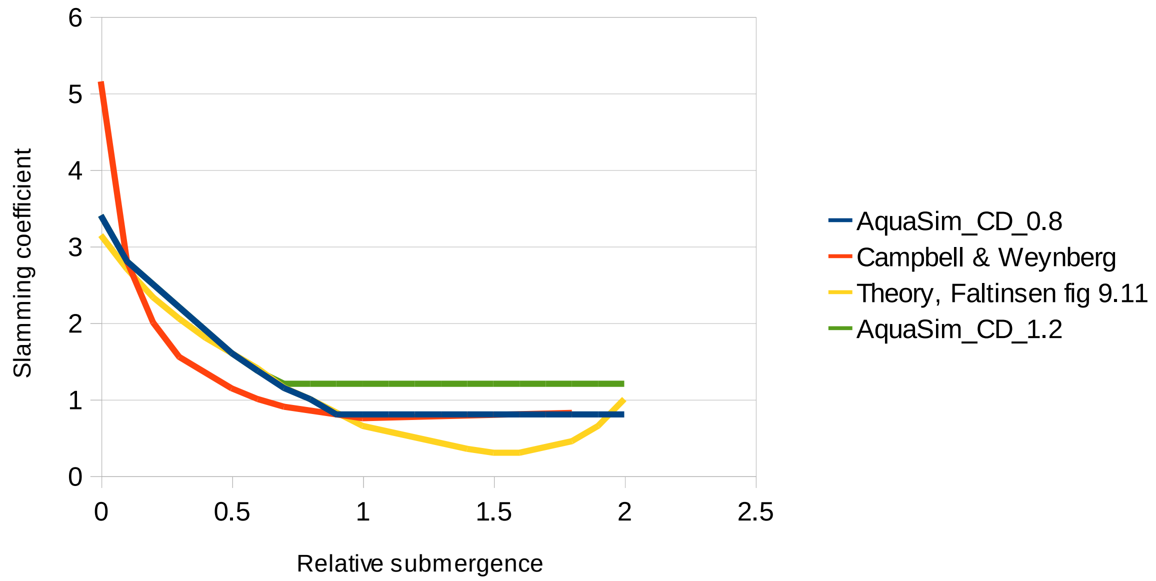

Figure 5

Submergence = 2 means fully submerged (2*radius from top)

Table 1 Slamming coefficients for sircular section

Submergence	AquaSim_CD_0.8
0	3.4
0.1	2.8
0.2	2.5
0.3	2.2
0.4	1.9
0.5	1.6
0.6	1.368
0.7	1.144
0.8	1
0.9	0.8
1	0.8
1.1	0.8
1.2	0.8
1.3	0.8
1.4	0.8
1.5	0.8
1.6	0.8
1.7	0.8
1.8	0.8
1.9	0.8
2	0.8

Note the default values are found by linear interpolation of the data seen in

Submergence = 2 means fully submerged (2*radius from top)

where AquaSim chooses the highest value of slamming coefficient and the drag coefficient to be used when the section is fully submerged.

The corresponding added mass values are the integral of the slamming coefficient meeting the fully submerged added mass coefficient when fully submerged.

Alternatively the user may introduce the table, **Feil! Fant ikke referansekilden.**, both for slamming coefficient and added mass coefficient as a function of submergence.

2.4.Slamming coefficients for rectangular sections

Alteratively to calculating slamming based on a circular section it may be calculated based on a rectangular section. In the latter case the relative height to with is found by considering the two drag diameters for drag.

The slamming parameters are in this case increased from values in Table 1 to values in Table 2 to the lowest 3 row. The rest is the same but in this case 1 means the top of the cross section (full height).

Table 2 Slamming coeff

Submergence, part of height	AquaSim_CD_0.8
0	6.8
0.05	5.04
0.1	3.25
0.15	2.2
0.2	1.9
0.25	1.6
0.3	1.368
0.35	1.144
0.4	1
0.45	0.8
0.5	0.8
0.55	0.8
0.6	0.8
0.65	0.8
0.7	0.8
0.75	0.8
0.8	0.8

0.85	0.8
0.9	0.8
0.95	0.8
1	0.8

3. WATER EXIT

At water exit the exact same calculation as for water entry is carried out. The only difference is that different values are used for $\frac{dA_{33}}{dh} = DL\frac{\rho}{2}Cd_{slam}$. The values for *Am* as a function of h for water exit are given as input, or default values in AquaSim are used. Note that also added mass in the local y- direction should be given as input (if not using AquaSim default values).

In water exit is defined as the case where the structure moves out of the water whereas water entry is defined as the case where the submerged part of a strip is increasing. At relative velocity close to 0, values are interpolated between the water exit and water entry values. This is when -0.1 m/s < V_{rel} < 0.1 m/s.

4. VALIDATION OF THE AQUASIM SLAMMING CALCULATION

2 cases have been analysed to validate the AquaSim slamming calculation. These are presented in the succeeding sections.

4.1. Case 1 beam through water at constant velocity

This case is shown in Figure 6 and Table 1. Results are seen in Figure 7. Results stems well.

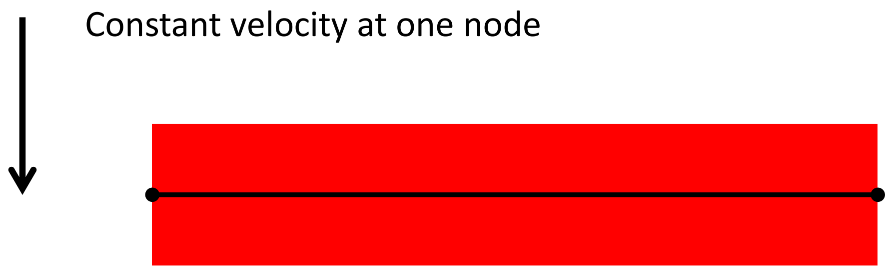

Figure 6 Horizontal beam moved at constant velocity through flat water line

Table 1 Case study particulars

Circular beam	
Diameter	0.5
Beam Length	10
Velocity	2
Drag diameter	0.5
Density beam	1025

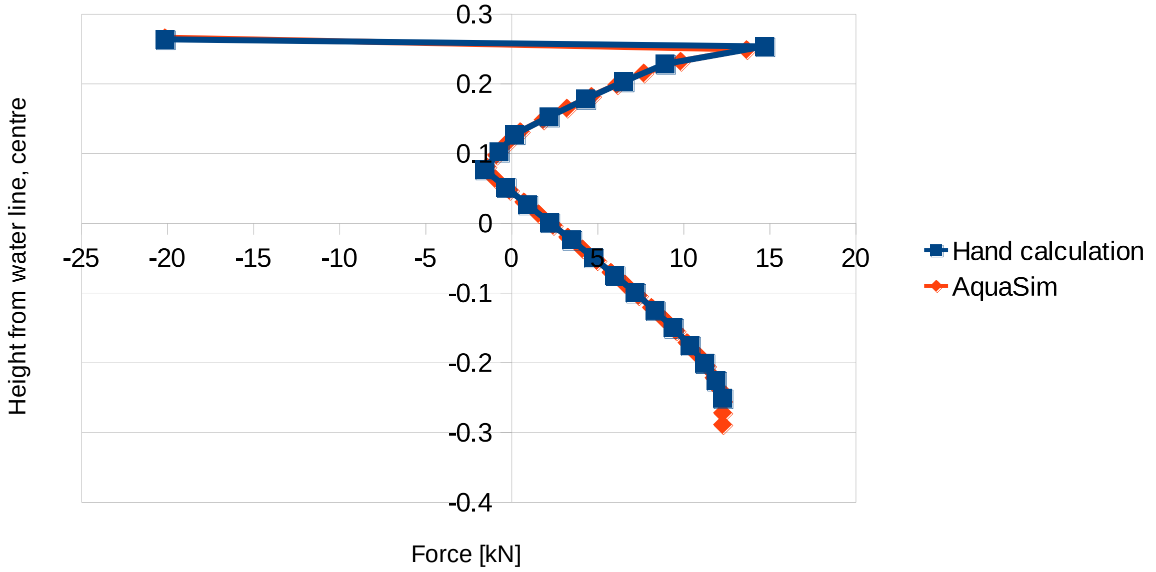

Figure 7 Comparison hand calculation to AquaSim

4.2. Fixed beam in waves

The beam shown in Figure 6 and Table 1 is kept still while a wave is applied to it directed in the local y- direction as shown in Figure 8. Figure 9 shows well comparing results

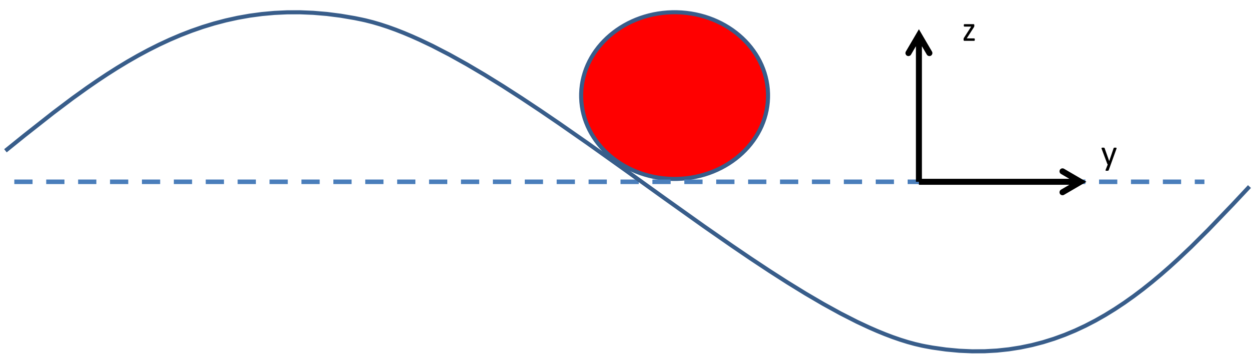

Figure 8 Fixed beam in waves

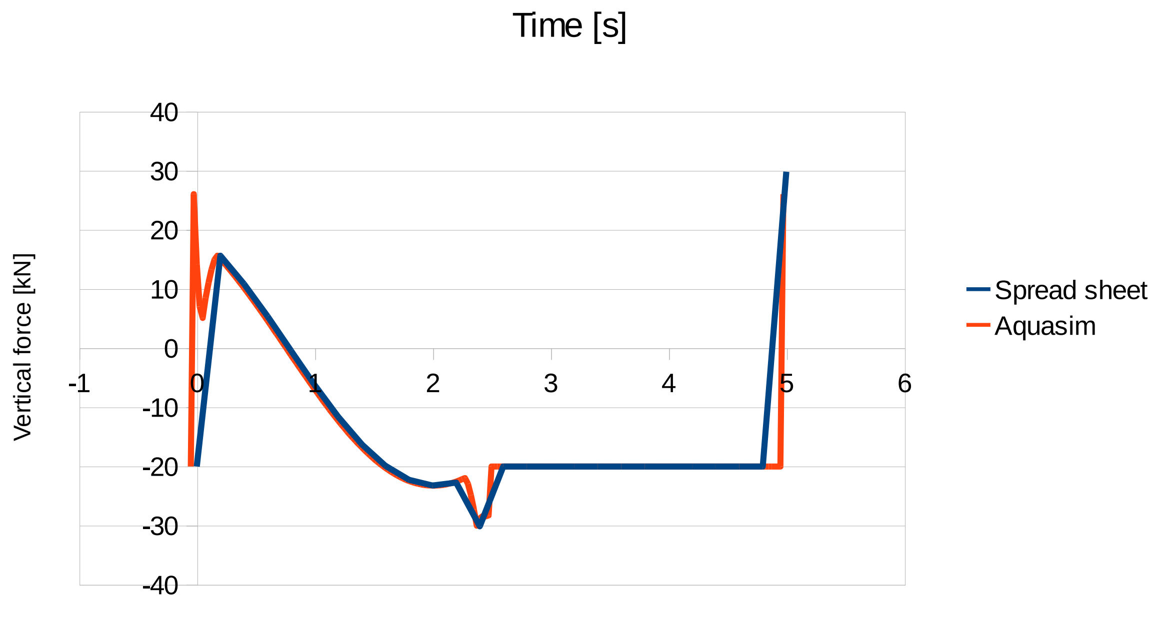

Figure 9

5. REFERENCES

Faltinsen, Odd M. (1990) “Sea loads on ships and offshore structures.” Cambridge university press ISBN 0 521 37285 2

Morison, J, M.O’Brien, J. Johnson and S. Shaff (1950). “The forces Exerted by surface waves on piles.” Petrolium Trans. 189, 149-1.

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